Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CCSS – Reg./Supple./Improv.) Examination, May 2014
CORE COURSE IN MATHEMATICS
6B11 MAT : Complex Analysis

Time : 3 Hours Max. Weightage : 30

***Instruction** : Answer to **all** questions.*

1. Fill in the blanks :

a) The principal argument of Arog (z) when $z = \frac{i}{-2-2i}$ = ____________

b) When z_2 and z_3 are non-zero complex numbers then

$\overline{\left(\frac{z_1}{z_2\,z_3}\right)}$ = ____________

c) $\left|\frac{z_1}{z_2\,z_3}\right|$ = ____________

d) $\overline{\overline{z}+3i}$ = ____________. (W = 1)

From questions **2** to **10** ; answer **any six**.

2. Prove that z is real if and only if $\overline{z} = z$.

3. Prove that $\overline{z_1 + z_2 + ... + z_n} = \overline{z}_1 + \overline{z}_2 + \overline{z}_3 + ... + \overline{z}_n$ for n = 2, 3, 4, ...

4. Find the exponential form of the complex number –1–i.

5. Find the derivative of f (x) = e^x (cosy + isiny).

6. Define an entire function. Give an example.

7. State Cauchy-Goursat theorem.

8. Prove that $f(z) = |z|^2$ is differentiable only at the origin.

9. If R is the radius of convergence of $\sum a_n z^n$, what is the radius of convergence $\sum a_n^2 z^n$?

10. Find the residue of $f(z) = \tan z$ at $z = \pi/4$. **(W = 6×1=6)**

From questions **11** to **20**; answer **any 7 :**

11. Verify Cauchy-Riemann equations for the function $f(z) = z^2$.

12. Show that $U = e^x (x \cos y - y \sin y)$ satisfies the Laplace's equation.

13. Prove the fundamental theorem of algebra.

14. Prove that a bounded entire function is a constant.

15. Show that an analytic function f (z) is a constant if its modulas is constant.

16. State Cauchy's Residue theorem.

17. Expand cosz about $z = \frac{\pi}{2}$ using Taylor's series.

18. What type of singularity have the $f(z) = \frac{1}{\sin z - \cos z}$ at $z = \pi/4$?

19. Find the residue of $f(z) = \frac{z^2}{z^2 + 4}$ at its poles.

20. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(ni)^2}{(2n)!} z^n$. **(W = 7×2=14)**

From questions **21** to **25**; answer **any 3 :**

21. Find the harmonic conjugate of the function $u(x, y) = y^3 - 3x^2y$.

22. State and prove Cauchy's integral formula.

23. Find two Laurent series expansions, in powers of z for the function

$$f(z) = \frac{1}{z(1+z^2)}.$$

24. When a singularity is said to be isolated ? What are different kinds of isolated singularities. Give example for each.

25. Prove that $\int_0^{2\pi} \frac{d\theta}{1 + a\cos\theta} = \frac{2\pi}{\sqrt{1-a^2}} \; (-1 < a < 1)$. **(W = 3×3=9)**

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