M 3137

Reg. No. : ....

Name : ....

VI Semester B.A./B.Sc./B.Com./B.B.A./B.B.A. T.T.M./B.B.M./B.C.A./ B.S.W./B.A. Afsal Ul Ulama Degree (CCSS – Reg./Supple./Improv.) Examination, May 2013

CORE COURSE IN MATHEMATICS

6B12 MAT : Linear Algebra

Time : 3 Hours Max. Weightage : 30

1. Fill in the blanks : **(Weightage 1)**

a) Dimension of $\mathbb{R}$ is ______________

b) Is every vector space have a basis ?

c) The system of linear equation AX = B is inconsistent if ______________

d) If λ is an eigen value of a matrix A, then the non-zero vector X satisfying the condition $AX = \lambda X$ is known as ______________.

Answer **any six** from the following : **(Weightage 1 each)**

2. Define linear dependence and independence of vectors.

3. Give an example for a vector space.

4. Prove that $W = \{(a, a, a) / a \in \mathbb{R}\}$ is a subspace of $V = \mathbb{R}^3$.

5. What do you mean by row reduced echelon form of a matrix ? Give an example.

6. Define eigen values and eigen vectors of a square matrix.

7. Prove that the matrices A and A^T have the same eigen values.

P.T.O.

8. Check whether the function $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x^2, y^2)$ is a linear transformation or not.

9. Define nilpotent operator.

10. Define rank of a matrix. **(Weightage 6×1=6)**

Answer **any seven** from the following : **(Weightage 2 each)**

11. Check whether $\mathbb{R}^2$ is a vector space over $\mathbb{R}$ or not with respect to the operations defined as follows :

$$(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

$$\alpha (x, y) = (0, 0)$$

12. Determine whether or not the vectors (1, 2, 5), (2, 5, 1) and (1, 5, 2) in $\mathbb{R}^3$ are linearly dependent.

13. Show that the equations $x + 2y - z = 3$, $3x - y + 2z = 1$, $2x - 2y + 3z = 2$, $x - y + z = -1$ are consistent and solve them.

14. Determine the values of λ for which the system of equations $3x + y - \lambda z = 0$; $4x - 2y - 3z = 0$; $2\lambda x + 4y + \lambda z = 0$ may possess non-trivial solution.

15. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$.

16. Prove that eigen values of a Skew-Hermitian matrix are either zero or purely imaginary.

17. Let U and V be vector spaces over the field F and let $T : U \to V$ be a linear transformation. Then T is one-to-one if and only if T maps linearly independent subsets of U onto linearly independent subsets of V.

18. Find the null space, range and their dimensions of the linear transformation $T : R^4 \to R^3$ defined by $T(x, y, z, w) = (x + y + w, z, y + 2w)$.

19. Let T be a linear operator defined on $\mathbb{R}^3$ such that $T(e_1) = e_1 + e_2$, $T(e_2) = e_1 - e_2 + e_3$, $T(e_3) = 3e_1 + 4e_3$ where $\{e_1, e_2, e_3\}$ is a standard basis for $\mathbb{R}^3$. Is T non-singular ? If so, find T^{-1}.

20. Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$. **(Weightage 7×2=14)**

Answer **any three** from the following : **(Weightage 3 each)**

21. Determine whether (1,1,1,1), (1, 2, 3, 2), (2, 5, 6, 4) and (2, 6, 8, 5) form a basis of $\mathbb{R}^4$. If not, find the dimension of the subspace they span.

22. Using row elementary transformations, find the inverse of the matrix

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

23. Using Cayley-Hamilton theorem, find the inverse of the matrix

$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}.$$

24. Diagonalise the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

25. Let $T : R^3 \to R^2$ be a linear transformation defined by $T(x, y, z) = (x + y, x + z)$. Find the matrix representation of T with respect to the ordered basis $X = \{(1, 0, 1), (1, 1, 0), (0, 1,1)\}$ in R^3 and $Y = \{(1, 3), (2, 5)\}$ in R^2.

(Weightage 3×3=9)