M 3136

Reg. No. : ....

Name : ....

VI Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W./ B.A. Afsal-Ul-Ulama Degree (CCSS – Reg./Supple./Improv.) Examination, May 2013

CORE COURSE IN MATHEMATICS

6B11 MAT : Complex Analysis

Time: 3 Hours Max. Weightage : 30

Instruction : *Answer to **all** questions.*

1. Fill in the blanks :

   a) If z_1 and z_2 are any two complex numbers then $|z_1+z_2| \geq$ ________

   b) $|z_1+z_2|^2 + |z_1 - z_2|^2 =$ ________

   c) $\arg(z) + \arg(-z) =$ ________ according as arg (z) is positive or negative.

   d) If $z_2, z_3 \neq 0$, then $\left|\frac{z_1}{z_2, z_3}\right| =$ ________

Questions **2** to **10** answer **any 6** from the following 9 questions. **(W – 1)**

2. When a function f(z) is said to be differentiable at z_0 ?
3. Define a harmonic function.
4. Find the square root of z = 26.
5. Find the principle argument Argz when $z = \left(\sqrt{3} - i\right)^6$
6. Prove that $f(z) = |z|^2$ is differentiable only at the origin.
7. State Cauchy's integral theorem.
8. If $f'(z) = 0$ everywhere in a domain D, then prove that f(z) must be a constant throughout D.
9. Find the radius of convergence of the series $\sum a_n z^{2n}$. If the radius of convergence $\sum a_n z^n$ is R.
10. Find the residue of f(z) = tanz at $z = \pi/2$ **(W. 6×1=6)**

P.T.O.

Questions **11** to **20**, Answer **any 7** from the following **10** questions :

11. Show that an analytic fun. f(z) u + iv is constant if its imaginary part is constant.
12. Show that $u = e^x (x \cos y - \sin y)$ satisfies Laplace's equation.
13. Evaluate $\int_C \frac{3z-1}{z^3 - z} dz$ where C is the circle $|z| = \frac{1}{2}$.
14. If f(z) is a polynomial of degree n ($n \geq 1$) with real or complex coefficients, then prove that f(z) = 0 has at least one complex root.
15. If f(z) is analytic within and on a circle C of radius r with centre at z_0, then prove that $|f^n(z_0)| \leq \mu \frac{n!}{r^n}$ for n = 0, 1, 2, ... where μ = max |f(z)| on C.
16. Expand $\frac{1}{z}$ by Taylor's series about z = 1.
17. State Cauchy's Residue theorem.
18. Evaluate $\int_C \frac{2z^2 + z}{z^2 - 1} dz$ where C is circle $|z-1| = 1$.
19. Prove that $f(z) = \sin\left(\frac{1}{z-a}\right)$ at z = a has an essential singularity.
20. Find out the zero's and discuss the nature of singularity of $f(z) = \left(\frac{z+1}{z^2+1}\right)^2$. **(W. 7×2=14)**

Questions **21** to **25**, answer **any 3** from the following **5** questions :

21. Establish the necessary conditions for a function f(z) to be analytic.
22. If f(z) is a regular function of z, then prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(u^2 + v^2)$
23. From the integral $\int_C \frac{dz}{z+2}$ where C is the circle |z|=1. show that $\int_0^{\pi} \frac{1 + 2\cos\theta}{5 + 4\cos\theta} d\theta = 0$.
24. State and prove Cauchy's integral formula.
25. Prove that $\int_0^{2\pi} \frac{d\theta}{5 + 3\cos\theta} = \pi/2$. **(W. 3×3=9)**

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