M 3135

Reg. No. : ....

Name : ....

VI Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W./
B.A. Afsal-Ul-Ulama Degree (CCSS – Reg./Supple./Improv.)
Examination, May 2013
CORE COURSE IN MATHEMATICS
6B10 MAT : Analysis and Topology

Time: 3 Hours Max. Weightage : 30

1. Fill in the blanks :

   a) If $P = \{ a = x_0, x_1, \ldots, x_n = b\}$ is a partition of [a, b], then the Riemann sum of a function $f : [a, b] \longrightarrow \mathbb{R}$ is ________________

   b) The radius of convergence of the series $\sum \frac{x^n}{n^2}$ is ________________

   c) The interior of the set of set of all rational numbers is ________________

   d) The limit point of the set $\left\{1, \frac{1}{2}, \frac{1}{3}, \ldots\right\}$ is ________________ **(Weight 1)**

2. If $f : [0, 6) \longrightarrow \mathbb{R}$ be defined by $f(x) = 4$ for all $x \in [0,6]$, show that f is integrable and evaluate integral.

3. Define the Riemann integral of a function $f : [a, b] \longrightarrow \mathbb{R}$.

4. If (f_n) is a sequence of functions defined on a subset D of $\mathbb{R}$ with values in $\mathbb{R}$, where $\mathbb{R}$ is the set of all real numbers, define the convergence of $\sum f_n$.

5. Show that $\lim\left(\frac{nx}{1+n^2x^2}\right) = 0$ for all $x \in \mathbb{R}$.

6. Prove that is a metric space each open sphere is an open set.

7. If X is a complete metric space and Y is a complete subspace of X, show that Y is closed.

8. If X is a metric space and $A \subset X$, prove that A is closed in X if and only if $A = \overline{A}$.

9. If T_1 and T_2 are two topologies on a non empty set X, prove that $T_1 \cap T_2$ is a topology on K.

10. If X is a topological space and A is a subset of X, show that $\overline{A} = A \cup D(A)$, where D(A) is the set of all limit points of A. **(6×1=6)**

Answer **any seven** from the following (Weight **2 each**) :

11. If $f(x) = x^2, x \in [0,4]$, calculate the Riemann sum if the portion P of [0, 4] is {0, 0.5, 2.5, 3.5, 4} with tags at the left end point of the intervals.

12. If $f : [a, b] \longrightarrow \mathbb{R}$ is monotone on [a, b], prove that $f \in R[a,b]$.

13. State and prove the Cauchy criterion for uniform convergence of a sequence $\{f_n\}$ for function on A, $A \subseteq \mathbb{R}$.

14. If (f_n) is a sequence of continuous functions on a set $A \subseteq \mathbb{R}$ and if (f_n) converges uniformly on A to a function $f : A \longrightarrow \mathbb{R}$, prove that f is uniformly continuous on A.

15. If R is the radius of convergence of the power series $\sum a_n x^n$, then prove that the series is absolutely convergent if $| x | < R$ and divergent in $| x | > R$.

16. If X is a non-empty set and d is a real function of ordered pairs of elements of X which satisfy the following two conditions :

    i) $d(x, y) = 0 \Leftrightarrow x = y$ and

    ii) $d(x, y) \leq d(x, z) + d(y, z)$, prove that d is a metric on X.

17. Prove that every non-empty open set on the real line is the union of a countable disjoint class of open intervals.

18. If a convergent sequence in a metric space X has infinitely many distinct points, prove that its limit is a limit point of the set of points of the sequence.

19. If X is a topological space and A is an arbitrary subset of X, prove that

    $\overline{A} = \{x : \text{each neighbourhood of } x \text{ intersects } A\}$.

20. Show that a subset of a topological space is perfect if and only if it is closed and has no isolated points. **(7×2=14)**

Answer **any three** from the following (Weight **3 each**) :

21 If $f \in R[a, b]$ and if f is continuous at a point $C \in [a,b]$, prove that the indefinite integral $F(x) = \int_a^z f, z \in [a,b]$ is differentiable at C and $F'(c) = f(c)$.

22. If (f_n) is a sequence of functions in R[a,b] and suppose (f_n) converges uniformly on [a, b], show that $f \in R[a,b]$.

23. If X is a metric space with metric d, prove that $d_1(x, y) = d(x, y)/1+d(x, y)$ is also a metric on X.

24. Show that is a metric space X :

    1) Any intersection of closed sets in X is closed and

    2) Finite union of closed sets in X is closed.

25. State and prove Cantor's intersection theorem. **(3×3=9)**

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