Reg. No. : ....

Name : ....

K20U 0128

VI Semester B.Sc. Degree (CBCSS-Reg./Supple./Improv.)
Examination, April 2020
(2014 Admission Onwards)
CORE COURSE IN MATHEMATICS
6B11MAT : Numerical Methods and Partial Differential Equations

Time : 3 Hours Max. Marks : 48

SECTION – A

All the **4** questions are **compulsory**. They carry **1** mark **each**.

1. State the intermediate value theorem for finding the real root of an equation.

2. Complete the expression $\Delta = E -$

3. Give the maximum bound for error $R_1(f)$ in trapezoidal rule.

4. For a function $u(r, \theta, t)$, give its Laplacian in Polar co-ordinates.

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Find $\sqrt{15}$ by Bisection method, correct to two decimal places.

6. Find a root of the equation $\log x - \cos x = 0$, where x is in radians, correct to two decimal places, using Regula Falsi method.

7. Show that $\Delta\left(\frac{f_i}{g_i}\right) = \frac{g_i \Delta f_i - f_i \Delta g_i}{g_i g_{i+1}}$.

P.T.O.

8. Find $\log_e(2.7)$ from the following table using Lagranges interpolation formula.

x	2	2.5	3
$\log_e(x)$	0.6932	0.9163	1.0986

9. Evaluate $\sqrt{\frac{2}{\pi}}\int_0^1 e^{-x^2/2}dx$, using Simpson's 1/3 rule, taking h = 0.25.

10. Evaluate $\int_0^1 \frac{dx}{1+x}$ using Trapezoidal rule with h = 0.25.

11. Find a solution to the initial value problem $y' = 2y - x$, $y(0) = 1$, by performing two iterations of the Picard's method.

12. Find y(1.2), given the differential equation $y' = -2xy^2$, with the condition $y(1) = 1$, using Taylor's series with step size h = 0.1.

13. Give the Fourier series solution of the one dimensional wave equation, with fixed ends and initial conditions $u(x, 0) = f(x)$ and $\frac{\partial u}{\partial t}|_{t=0} = g(x)$.

14. Solve the equation $u_{yy} = 0$ where u is a function of x and y.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. Find a real root of the equation $x^3 + x^2 - 1 = 0$ by General iteration method, correct to two decimal places.

16. Using Newtons divided difference formula, find a cubic polynomial for the following data. Hence find f(3).

x	0	1	2	4
f(x)	1	1	2	5

17. The function f(x) represented by the following data has a minimum in the interval (0.5, 0.8). Find this point of minimum and the minimum value.

x	0.5	0.6	0.7	0.8
f(x)	1.3254	1.1532	0.9432	1.0514

18. Find the approximate value of y(0.1) given that $y' = x^2 + y^2$, y(0) = 1 using three iterations of the Modified Euler's method with h = 0.1.

19. Given $\frac{dy}{dx} = y - x$ with y(0) = 2, use Runge – Kutta method of order two to find y(0.2) taking h = 0.1.

20. A stretched string of length l and fixed end points has initial displacement $y = a \sin \frac{\pi x}{l}$ from which it is released at time t = 0. Find the vertical displacement y(x, t) at any distance x from one end at time t.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. Find an interval of unit length which contains the smallest positive root of the equation $e^x - 2x^2 = 0$. Hence find the root of this equation using Newton – Raphson method correct to three decimal places.

22. The following table gives the value of e^{-x} for some values of x :

x	0.2	0.3	0.4	0.5	0.6	0.7	0.8
e^{-x}	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493

Determine the value of $e^{-0.55}$ using Stirling's central difference formula.

23. Compute f′(0.2) and f″(0) from the following table.

x	0.0	0.2	0.4	0.6	0.8	1.0
f(x)	1.00	1.16	3.56	13.96	41.96	101.00

24. Find the temperature u(x, t) in a slab of length L whose ends are kept at zero temperature and whose initial temperature f(x) is given by

$$f(x) = \begin{cases} k, & \text{when } 0 < x < \frac{L}{2} \\ 0, & \text{when } \frac{L}{2} < x < L \end{cases}$$