Reg. No. : ....

Name : ....

K20U 0130

VI Semester B.Sc. Degree (CBCSS-Reg./Supple./Improv.) Examination, April 2020
(2014 Admission Onwards)
Core Course in Mathematics
6B13MAT : MATHEMATICAL ANALYSIS AND TOPOLOGY

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. If $I = [0, 4]$, calculate the norm of the partition $\mathcal{P} = \{0,1, 1.5, 2, 3.4, 4\}$.
2. Evaluate $\lim (f_n(x))$ where $f_n(x) = \frac{x}{x+n}$ for all $x \geq 0$, $n \in \mathbb{N}$.
3. Fill in the blanks : The closure of set of all irrational numbers is ________
4. Write a pair of topologies T_1 and T_2 on $X = \{a, b, c\}$ so that $T_1 \cup T_2$ is not a topology on X.

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Show that every constant real valued function on [a, b] is in $\mathcal{R}[a, b]$.
6. State squeeze theorem for Riemann integrability.
7. Find the value of $\int_{-10}^{10} \text{sgn}(x)\, dx$.
8. Prove that the sequence of functions, $f_n(x) = \frac{x}{n}$, $n \in \mathbb{N}$ converges uniformly on [0, 1]
9. State the Bounded Convergence Theorem.
10. Define a metric space and write an example.
11. Prove that in a metric space each open sphere is an open set.
12. Give an example of a pair of subsets A and B of the real line with usual topology such that $\text{Int}(A) \cup \text{Int}(B) \neq \text{Int}(A \cup B)$.
13. Define subspace of a topological space and show that it is a topological space.
14. Is the real line $\mathbb{R}$ with the usual topology separable ? Justify.

P.T.O.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. Prove that if $f : [a, b] \to \mathbb{R}$ is continuous on [a, b], then $f \in \mathcal{R}[a, b]$.

16. State and prove composition theorem in Riemann integrals. Deduce that if $f \in \mathcal{R}[a, b]$, then $|f| \in \mathcal{R}[a, b]$ and $\left|\int_a^b f\right| \leq \int_a^b |f|$.

17. Prove that a power series $\sum a_n x^n$ is absolutely convergent if $|x| < R$ and is divergent if $|x| > R$. (Here R is the radius of convergence and assume that $0 < R < \infty$).

18. Show that a subset F of a metric space is closed if and only if its complement F' is open.

19. Prove that a subspace Y of a complete metric space X is complete if and only if it is closed.

20. Prove that in a topological space $\overline{A} = A \cup D(A)$ and A is closed if and only if $A \supseteq D(A)$.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. State and prove the Cauchy criterion for Riemann integrability.

22. Prove that if (f_n) is a sequence of functions in $\mathcal{R}[a, b]$ and (f_n) converges uniformly on [a,b] to f, then $f \in \mathcal{R}[a, b]$ and $\int_a^b f = \lim_{n \to \infty} \int_a^b f_n$.

23. Show that in a complete metric space X, if $\{F_n\}$ is a decreasing sequence of non-empty closed subsets of X such that $d(F_n) \to 0$, then $\cap_{n=1}^{\infty} F_n$ contains exactly one point. Give an example to show that the condition $d(F_n) \to 0$ can not be dropped to obtain the result.

24. Show that a subset of a topological space is dense if and only if it intersects every non-empty open set.