K20U 0127

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS-Reg./Supple./Improv.) Examination, April 2020
(2014 Admission Onwards)
CORE COURSE IN MATHEMATICS
6B10MAT : Linear Algebra

Time : 3 Hours Max. Marks : 48

SECTION – A

All the **first 4** questions are **compulsory**. They carry **1** mark **each**.

1. Give an example to show that if f and g are two quadratic polynomials then the polynomial f + g need not be quadratic.

2. Obtain a basis for $M_{2\times 2}$ (R).

3. Let V = P_2 (R) and let $\beta = \{1, x, x^2\}$ be the standard ordered basis for V. If $f(x) = 3x^2 + 2x + 1$ then $[f]_\beta$ is

4. Give the nature of characteristic roots of
   i) a Hermitian matrix and
   ii) a Unitary matrix.

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Find the equation of the line through the points P (2, 0, 1) and Q (4, 5, 3).

6. What is the possible difference between a generating set and a basis ?

7. Is the union of two subspaces W_1 and W_2 of a vectorspace V again a subspace of V ? Justify with an example.

P.T.O.

8. Let V be a vectorspace and $\beta = \{x_1, x_2, ..., x_n\}$ be a subset of V. Show that β is basis if each vector y in V can be uniquely expressed as a linear combination of vectors in β.

9. Show that $T : R^2 \to R^2$ defined by $T(a_1, a_2) = (2a_1 + a_2, a_1)$ is a linear transformation.

10. Let $T : V \to W$ be a linear transformation. Prove that N (T), the nullspace of T, is a subspace of V.

11. Find a basis of the row space of the matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{bmatrix}.$$

12. Find the characteristic values of the matrix

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}.$$

13. Use Gauss elimination to solve the system of equations :

$10x + y + z = 12$

$2x + 10y + z = 13$

$x + y + 3z = 5.$

14. Use Gauss, Jordan elimination to solve the system of equations :

$10x + y + z = 12$

$2x + 10y + z = 13$

$x + y + 3z = 5.$

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. In every vectorspace V over a field F prove that

    i) $a0 = 0 \ \forall a \in F$, where 0 is the zero vector and

    ii) $(-a)x = -(ax) \ \forall a \in F$ and $\forall x \in V$.

16. Define linear dependence and linear independence of vectors with examples.

17. Define a linear transformation from a vectorspace V into W. Verify that $T : M_{m \times n} \to M_{n \times m}$ by $T(A) = A^t$ where A^t is the transpose of A, is linear.

18. Show that the row nullity and column nullity of a square matrix are equal.

19. Find the characteristic values and the corresponding characteristic vectors of the matrix.

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

20. Use the Gaussian elimination method to find the inverse of the matrix.

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$$

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. If a vectorspace V is generated by a finite set S_0, then show that a subset of S_0 is a basis for V and V has a finite basis.

22. State and prove dimension theorem. Deduce that a linear transformation $T : V \to V$ is one to one if and only if T is onto.

23. Show that the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$$

satisfies Cayley Hamilton theorem and hence obtain A^{-1}.

24. Prove that

$$A = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}$$

is diagonalizable and find the diagonal form.
