K20U 0129

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS – Reg./Supple./Improv.)
Examination, April 2020
(2014 Admission Onwards)
CORE COURSE IN MATHEMATICS
6B12MAT : Complex Analysis

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. Sketch the region $\{z : \text{Re}\,(iz) \geq 0\}$.
2. Define Harmonic function.
3. Find the Radius of convergence of $\sum 7^n z^n$.
4. Find the residue of $f(z) = e^z$ at $z = 0$.

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Give an example of a function which is differentiable exactly at one point and give its justification.
6. Verify Cauchy-Riemann equations for the function $f(z) = z^2$.
7. Evaluate $\int_C |z|\,dz$, where C is the line segment from origin to 1 + i.
8. Find the Radius of convergence of $\sum (1 + i)^n (z - 3i)^n$.
9. Find the residue of $f(z) = \dfrac{9z + i}{z(z^2 + 1)}$ at $z = i$.

P.T.O.

10. Find the Laurent's series expansion of $f(z) = \frac{1}{z^5}\sin z$ with center 0.

11. State Taylors Theorem. Find the Taylors series expansion of $f(z) = \frac{1}{1+z^2}$ centered at $z = 0$.

12. Give an example of a series which is convergent but not absolutely. Give justification.

13. State Laplace's Equation. Give an example of a real valued function which satisfy Laplace's Equation on the complex plane.

14. State Cauchy's inequality.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry 4 marks **each**.

15. Prove that an analytic function of constant absolute value is constant in a domain.

16. Evaluate the following :

 a) $\int_0^{1+i} z^2 dz$

 b) $\int_{8+\pi i}^{8-3\pi i} e^{\frac{z}{2}} dz$

17. The power series $\sum a_n z^n$ converge at $z = 1$ and diverge at $z = -1$. Find the radius of convergence of $\sum a_n z^n$.

18. State and prove Residue Theorem.

19. Find an analytic function $f(z) = u(x, y) + iv(x, y)$, where $u(x, y) = xy$.

20. State and prove the theorem of convergence of power series.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. State and prove Cauchy – Riemann equations.

22. a) Define singular point, isolated singular point, removable singular point, pole and essential singular point.

    b) Give an example of a non-isolated singular point.

23. a) State and prove Cauchy's integral formula.

    b) Evaluate $\int_C \frac{e^z}{z-2}\,dz$, where C is the circle $|z| = 3$.

24. Give examples and justifications of power series having Radius of convergence 1 and

    a) Which diverge at every point on the circle of convergence ?

    b) Which doesn't diverge at every point on the circle of convergence ?