K19U 0122

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS-Reg./Supple./Improv.) Examination, April 2019
(2014 Admission Onwards)
CORE COURSE IN MATHEMATICS
6B10 MAT : Linear Algebra

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. Give an example of a proper non trivial subspace of P(R), the vectorspace of all polynomials with real coefficients.

2. A subset of a linearly dependent set can possibly be linearly independent. Justify by giving an example.

3. The null space of the operator $T : R^2 \mapsto R^2$ given by $T(a_1, a_2) = (a_1, 0)$ is

4. The number of linearly independent solutions of the equation $x + y + z = 0$ is

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. In any vectorspace V show that $(a + b)(x + y) = ax + bx + ay + by$ for all scalars a and b and all vectors x and y.

6. Let $V = R^2 = R \times R$ where vector addition and scalar multiplication are defined by; $(x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2)$ and $r(x_1, x_2) = (rx_1, x_2)$. Is V a vectorspace over R ? Justify.

7. Show that any intersection of subspaces of a vectorspace V is again a subspace of V.

8. Let $T : V \mapsto W$ be a linear transformation. Prove that N(T), the nullspace of T, is a subspace of V.

9. Let $T : V \mapsto W$ be an invertible linear transformation. Use dimension theorem to observe that dimV = dimW.

10. Let V = C[a,b] be the vectorspace of all continuous real valued functions defined over the closed bounded interval [a, b]. Describe the fundamental theorem of calculus interms of linear transformations on V.

11. Find the values of λ for which the following system of equations have non zero solutions.
$\lambda x + 8y = 0$
$2x + \lambda y = 0$

12. Verify that the set of all characteristic vectors of a square matrix associated with a fixed characteristic value λ is a subspace of the respective Eucledian space.

13. Use Gauss elimination to solve the system of equations :
$2x + y + z = 10$
$3x + 2y + 3z = 18$
$x + 4y + 9z = 16$

14. Use Gauss Jordan elimination to solve the system of equations :
$2x + y + z = 10$
$3x + 2y + 3z = 18$
$x + 4y + 9z = 16$

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each.**

15. Define vectorspace and show that in every vector space (–1) x is the additive inverse of x.

16. Define a basis of a vectorspace. Give an example of a basis of $M_{2\times 2}(R)$.

17. Let V and W be vectorspaces and let T : V $\mapsto$ W be linear. Then prove that T is one to one if and only if N(T) = {0}.

18. Suppose that AX = B has a solution. Show that this solution is unique if and only if AX = 0 has only the trivial solution.

19. Test the following system of equations for consistency and solve it if it is consistent.

$x + 2y + 3z = 14$

$3x + y + 2z = 11$

$2x + 3y + z = 11$

20. Find the largest characteristic value and a corresponding characteristic vector of the matrix.

$$\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. If S is a nonempty subset of a vectorspace V, then show that span (S) is a subspace of V and is the smallest subspace of V containing S. Under what further condition S can become a basis of V ?

22. Let V and W be vectorspaces over a common field F and suppose that V has a basis $\{x_1, x_2, \ldots, x_n\}$. Prove that for any fixed vectors $y_1, y_2, \ldots, y_n$ in W there exists exactly one linear transformation $T : V \mapsto W$ such that $T(x_i) = y_i$ for $i = 1 \ldots n$.

23. Show that the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$$

satisfies Cayley Hamilton theorem and hence obtain A^{-1}.

24. Prove that

$$A = \begin{bmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

is diagonalizable and find the diagonal form.