M 485

Reg. No. : ....

Name : ....

VI Semester B.A./B.Sc./B.Com./B.B.A./B.B.A. T.T.M./B.B.M./B.C.A./B.S.W./ B.A. Afsal Ul Ulama Degree (CCSS-Regular) Examination, April 2012

CORE COURSE IN MATHEMATICS

6B12 MAT : Linear Algebra

Time : 3 Hours Max. Weightage : 30

1. Fill in the blanks :

   a) Dimension of $P_n(t)$ of all polynomials of degree less than or equal to n with real coefficients is ______________

   b) Is every vector space have a basis ?

   c) Roots of characteristic equation of a square matrix is known as ______________

   d) If a matrix A is in row-reduced echelon form, then the rank of A is ___________

(Weightage 1)

Answer **any six** from the following : **(Weightage 1 each)**

2. Define basis of a vector space.

3. Prove that every subset of a linearly dependent set is linearly dependent.

4. Prove that the set of all symmetric matrices of order n is a subspace of the vector space of all square matrices of order n.

5. Explain the conditions for the consistency and number of solutions of non-homogeneous linear system of equations AX = B.

6. If λ is an eigen value of A, prove that λ^n is an eigen value of A^n.

7. State Cayley Hamilton theorem.

P.T.O.

8. Check whether the function $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x, y, z) = (x + 1, y + z)$ is a linear transformation or not.

9. Define null space and range space of a linear transformation.

10. Define inverse of a linear transformation. **(Weightage 6×1=6)**

Answer **any seven** from the following : **(Weightage 2 each)**

11. Show that $\{(1, 0, 0)\ (0, 1, 0), (1, 1, 1)\}$ is a basis for $\mathbb{R}^3$.

12. Check whether or not $u = 2t^2 + 4t - 3$ and $v = 4t^2 + 8t - 6$ are linearly dependent.

13. Test for consistency and solve the equations $x + y + z = 6$, $x + 2y + 3z = 14$, $x + 4y + 7z = 30$.

14. Determine the values of λ for which the system of equations $3x + y - \lambda z = 0$; $4x - 2y - 3z = 0$; $2\lambda x + 4y + \lambda z = 0$ may possess non-trivial solution.

15. Find the eigen values of $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

16. Prove that eigen vectors corresponding to distinct eigen values of a matrix are linearly independent.

17. Let $T : U \to V$ be a linear map. Prove that T is one-to-one if and only if null space of T is $\{0\}$.

18. Find the null space, range and their dimensions of the linear transformation $T : R^3 \to R$ defined by $T(x, y, z) = x + 3y$.

19. Let T be a linear operator on P_2, the set of all polynomials of degree ≤ 2, defined by $T(a_0 + a_1x + a_2x^2) = a_0 + (a_1 - a_2)\,x + (a_0 + a_1 + a_2)x^2$. Find T^{-1}.

20. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$. **(Weightage 7×2=14)**

Answer **any three** from the following : **(Weightage 3 each)**

21. Check whether $\mathbb{R}^n$ is a vector space over $\mathbb{R}$ or not with respect to the operations defined as follows. $(x_1, x_2, \ldots x_n) + (y_1, y_2, \ldots, y_n) = (x_1 + y_1, x_2 + y_2, \ldots, x_n + y_n)$ $\alpha(x_1, x_2, \ldots, x_n) = (\alpha x_1, \alpha x_2, \ldots, \alpha x_n)$.

22. Using row elementary transformations, find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & -1 \\ -2 & 1 & 3 \end{bmatrix}$.

23. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$.

24. Diagonalise the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

25. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by $T(x, y, z) = (0, x+y, x+y+z)$. Find the matrix representation of T with respect to the ordered basis $X = \{(1, 0, 1), (1, 1, 0), (0, 1, 1)\}$ in $\mathbb{R}^3$ and $Y = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)$ in $\mathbb{R}^3$.

(Weightage 3×3=9)
