



M 486

Reg. No. : .....

Name : .....

**VI Semester B.A./B.Sc./B.Com./B.B.A./B.B.A. T.T.M/B.B.M./B.C.A./B.S.W./  
B.A. Afsal UI Ulama Degree (CCSS – Regular) Examination, April 2012  
CORE COURSE IN MATHEMATICS  
6B13 MAT : Integral Transforms**

Time : 3 Hours

Max. Weightage : 30

1. Fill in the blanks :

a) Laplace transform of  $t^n$  ( $n$  is an integer) is \_\_\_\_\_

b) Period of  $\sin x$  is \_\_\_\_\_

c) A function  $f(x)$  is said to be even if  $f(-x) =$  \_\_\_\_\_

d)  $Z(\delta(n)) =$  \_\_\_\_\_

(Weightage 1)

Answer **any six** from the following (Weightage **1 each**) :

2. State first shifting theorem for Laplace transform.

3. Find  $L(t \sin at)$ .

4. Find the inverse Laplace transform of  $\frac{s}{(s+1)^2}$ .

5. Explain the Euler formula for Fourier series expansion.

6. Define Z-transform.

7. Find Z-transform of  $\cos n\theta$ .

8. Find  $Z^{-1}\left\{\frac{1}{(z+1)}\right\}$ .

9. Explain Fourier sine and cosine integral representations of functions.

10. Find Fourier sine transform of  $f(x) = \begin{cases} k & 0 < x < a \\ 0 & x > a \end{cases}$  (Weightage  $6 \times 1 = 6$ )

P.T.O.



Answer **any seven** from the following (Weightage **2 each**) :

11. Define unit step function. Also find its Laplace transform.

12. Find the inverse Laplace transform of  $\frac{2}{s^2} - \frac{2e^{-2s}}{s^2} - \frac{4e^{-4s}}{s} + \frac{se^{-\pi s}}{s^2 + 1}$ .

13. Find the Fourier series expansion of  $f(x) = x + x^2$  from  $x = -\pi$  to  $x = \pi$ .

14. Express  $f(x) = x$  as a half range cosine series in  $0 < x < 1$ .

15. Find the complex Fourier series of  $f(x) = e^x$ ,  $-\pi < x < \pi$  and  $f(x + 2\pi) = f(x)$ .

16. Find Z-transform of  $\frac{1}{n(n+1)}$ .

17. Find the Z-transform of  $f * g$  where  $f(n) = u(n)$  and  $g(n) = 2^n u(n)$ .

18. Using partial fraction method, find the inverse Z-transform of  $\frac{2z^2 + 3z}{(z+2)(z-4)}$ .

19. Find the Fourier cosine integral of  $f(x) = e^{-kx}$ .

20. With usual notation, prove that  $F_c\{f''(x)\} = -w^2 F_c\{f(x)\} - \sqrt{\frac{2}{\pi}} f'(0)$ . (Weightage  $7 \times 2 = 14$ )

Answer **any three** from the following (Weightage **3 each**) :

21. State and prove Convolution theorem for Laplace transforms.

22. Using Laplace transform, solve the initial value problem :

$$y'' + 3y' + 2y = 8 \cos 2t, y(0) = -1, y'(0) = 2.$$

23. Obtain the Fourier series for the function  $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi}, & 0 \leq x \leq \pi \end{cases}$ .

$$\text{Deduce that } \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}.$$

24. a) Using convolution method, find the inverse Z-transform of  $\frac{z^2}{(z-a)(z-b)}$ .

b) Using long division method, find the inverse Z-transform of  $\frac{10z}{z^2 - 3z + 2}$ .

25. Find the Fourier transform of  $e^{-ax^2}$ ,  $a > 0$ .

(Weightage  $3 \times 3 = 9$ )