M 484

Reg. No. : ....

Name : ....

VI Semester B.A./B.Sc./B.Com./B.B.A./B.B.A. T.T.M./B.B.M./B.C.A./B.S.W./ B.A. Afsal Ul Ulama Degree (CCSS – Regular) Examination, April 2012
CORE COURSE IN MATHEMATICS
6B11 MAT : Complex Analysis

Time: 3 Hours Max. Weightage : 30

***Instruction :** Answer **all** questions.*

1. Fill in the blanks :

a) For any two complex numbers z_1 and z_2 $|z_1 + z_2| \leq$ ____________

b) $|z_1 - z_2| \geq$ __________

c) $\left|\frac{z_1}{z_2 z_3}\right| =$ ___________ when z_2 and z_3 are non zero.

d) $z\overline{3} =$ __________ **(W-1)**

Questions **2** to **10**. Answer **any 6** from the following **9** questions.

2. Write the principal argument of the complex number $-1 - i$ which lies in the 3[rd] quadrant.

3. Find the square root of the complex number $z = 1 - \sqrt{3}i$.

4. Define a harmonic function.

5. Prove that $f(z) = |z|^2$ is differentiable only at the origin.

6. Find the values of z for which $e^z = -1$.

P.T.O.

7. Define the principal branch of Log z.

8. State Cauchy-Goarsat theorem.

9. When a series $\sum a_n z^n$ is said to be conditionally convergent ?

10. What is the nature of singularity for e^z at $z = \infty$? **(W-6×1=6)**

Questions **11** to **20**. Answer **any 7** from the following **10** questions.

11. Verify Cauchy-Riemann conditions for the following function

$$f(z) = \frac{x - iy}{x^2 + y^2}.$$

12. Show that an analytic function $f(z) = u + iv$ is constant if its real part is constant.

13. Evaluate $\int_C \frac{dz}{z - a}$ where C is the circle $|z - a| = r$.

14. State and prove Liouville's theorem.

15. If f(z) is a polynomial of degree u $(u \geq 1)$ with real or complex coefficients then prove that the equation $f(z) = 0$ has at least one complex root.

16. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} z^n$.

17. Prove that the function $f(z) = \frac{\sin(3 - z_0)}{z - z_0}$ has a removable singularity at $z = z_0$.

18. Find the zeros and discuss the nature of singularity of $f(z) = \frac{z - 2}{z^2} \sin\left(\frac{1}{z - 1}\right)$.

19. Find the residues of $\frac{z+1}{z^2(3-2)}$ at its poles.

20. Evaluate the integral $\int_C \frac{5z-2}{z(3-1)} dz$ where C is circle $|z| = 2$ described counter clockwise.

(W-7×2=14)

Questions **21** to **25**. Answer **any 3** from the following **5** questions :

21. Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, even though Cauchy-Riemann equations are satisfied at that point.

22. Show that $u = y^3 - 3x^2y$ is a harmonic function. Find its conjugate.

23. State and prove Cauchy's integral formula.

24. If f(z) is analytic inside and on a closed contour C and z_0 is any point inside C, then prove that $f'(z_0) = \frac{1}{2\pi i} \int_C \frac{f(z)}{(3-z_0)^2} dz$.

25. Show that $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta} = \frac{2\pi}{\sqrt{3}}$.

(W-3×3=9)
