Reg. No. : ....

Name : ....

M 483

VI Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W./ B.A. Afsal Ul Ulama Degree (CCSS-Regular) Examination, April 2012

CORE COURSE IN MATHEMATICS

6B10 MAT : Analysis and Topology

Time : 3 Hours Max. Weightage : 30

1. Fill in the blanks :

   a) The norm of a partition of an interval is defined as ____________

   b) The radius of convergence of the series $\sum x^n/n$ is ____________

   c) The limit point of the set $\left\{1, 1/2, 1/3 ....\right\}$ is ____________

   d) A subset A of a topological space X is said to be dense if ____________

(Weightage 1)

Answer **any six** from the following. Weight **1 each**.

2. If $f : [0, 6] \rightarrow R$ be defined by $f(x) = 4$ for all $x \in [0, 6]$, show that f is integrable and evaluate its integral.

3. State the first form of fundamental theorem of integral calculus.

4. Define convergence and uniform convergence of a sequence of functions.

5. Show that $\lim \left(nx/_{1 + n^2x^2} \right) = 0$ for all $x \in IR$.

6. Prove that $d(x, y) = | x - y |$ is a metric on IR.

7. Prove that a closed subspace of a complete metric space is complete.

8. Prove that in a metric space X, the complement of a closed set is open.

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9. If T_1 and T_2 are two topologies on a non-empty set X, prove that $T_1 \cap T_2$ is also a topology on X.

10. If X is a topological space and A is a subset of A, show that $\overline{A} = A \cup D(A)$, where D(A) is the set of all limit points of A. **(Wt. 6×1=6)**

Answer **any seven** from the following (Weight **2 each**) :

11. If $f(x) = x^2$, $1 \le x \le 3$ and $P = \{1, 1.5, 2.1, 2.6, 3\}$ is a portion of [1, 3], calculate the Riemann sum of f corresponding to the tags given by $t_k \in \{1, 2, 2.5, 2.7\}$.

12. If $f \in R[a, b]$, prove that f is bounded on [a, b].

13. Prove that a sequence (f_n) of bounded functions on $A \subseteq \mathbb{R}$ converges uniformly on A to f if and only if $\| f_n - f \|_A \to 0$.

14. If (f_n) is a sequence of continuous functions on a set $A \subseteq \mathbb{R}$ and if (f_n) converges uniformly on A to a function $f : A \to \mathbb{R}$, then prove that f is continuous on A.

15. If (f_n) is a monotone sequence of continuous functions on I = [a, b] that converges on I to a continuous function f, then prove that the convergence of the sequence is uniform.

16. If X is a non-empty set and if d is a real valued function of ordered pairs of elements of X which satisfies the following two conditions d(x, y) = 0 if and only if x = y and $d(x, y) \le d(x, z) + d(y, z)$, then prove that d is a metric on X.

17. Prove that every convergent sequence in a metric space X is a Cauchy sequence.

18. Show that if $\{A_n\}$ is a sequence of nowhere dense sets in a complete metric spaceX, then there exists a point in X which is not in any of the A_n's.

19. If X is a topological space and A is an arbitrary subset of X, prove that $\overline{A} = \{x :$ each neighbourhood of x, intersects $A\}$.

20. Show that a subset of a topological space is perfect if and only if it is closed and has no isolated points. **(Wt. 7×2=14)**

Answer **any three** from the following (Weight **3 each**) :

21. If $f : [a, b] \to \mathbb{R}$ is continuous on [a, b], prove that $f \in R[a, b]$.

22. If (f_n) is a sequence of functions in R [a, b] and suppose that (f_n) converges uniformly on [a, b], then show that $f \in R[a, b]$.

23. If X is a metric space with metric d, prove that $d_1(x, y) = {d(x, y)}/{1 + d(x, y)}$ is also a metric on X.

24. Show that in a metric space X :
    1) any union of open sets is open and
    2) any finite intersection of open sets is open.

25. If $f : X \to Y$ is a mapping of one topological space into another, show that the following conditions are equivalent.
    a) f is continuous
    b) for every subset A of X; $f(\overline{A}) \subseteq \overline{f(A)}$
    c) for every closed set B in Y, $f^{-1}(B)$ is closed in X. **(Wt. 3×3=9)**

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