0058646 **K19U 2254**

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS-Reg./Sup./Imp.) Examination, November - 2019
(2014 Admn. Onwards)
Core Course in Mathematics
5B05 MAT : REAL ANALYSIS

Time : 3 Hours Max. Marks : 48

SECTION - A

Instructions: Answer **all** questions. Each question carries **One** mark. **(4×1=4)**

1. State Arithmetic-Geometric Mean Inequality.
2. Find $\sup\left\{\frac{1}{m}-\frac{1}{n} : n \in \mathbb{N}\right\}$.
3. Show that the sequence $\left(\frac{1}{n}\right)$ is a Cauchy sequence.
4. State Weierstrass Approximation theorem.

SECTION - B

Answer any **Eight** questions. Each carries **Two** marks. **(8×2=16)**

5. State and prove triangle inequality.
6. Show that the sequence (2^n) does not converges.
7. Show that the series $\sum_{n=1}^{\infty} \cos n$ is divergent.
8. Show that if a convergent series contains only a finite number of negative terms, then prove that it is absolutely convergent.
9. Let $X = (x_n)$ be a nonzero sequence in $\mathbb{R}$ and let $a = \lim\left(n\left(1-\left|\frac{x_{n-1}}{x_n}\right|\right)\right)$, whenever the limit exists. Then prove that $\sum x_n$ is absolutely convergent when $a > 1$ and is not absolutely convergent when $a < 1$.

P.T.O.

10. What do you mean by saying that a function is continuous at a point c.

11. Let $A \subset \mathbb{R}$, let f and g be functions on A to $\mathbb{R}$, and if $g(x) \neq 0$ for all $x \in \mathbb{R}$. Suppose that $c \in A$ and that f and g are continuous at c. Then show that f/g is continuous at c.

12. Give an example of two functions f and g that are both discontinuous at a point c in $\mathbb{R}$ such that the sum $f + g$ is continuous at c.

13. If $f : A \to \mathbb{R}$ is a Lipschitz function, then prove that f is uniformly continuous on A.

14. Let $I \subset \mathbb{R}$ be an interval and $f : I \to \mathbb{R}$ be increasing on I. If $c \in I$ then prove that f is continuous at c if and only if $j_f(c) = 0$, where $j_f(c)$ is the jump of f at c.

SECTION - C

Answer any **Four** questions. Each carries **Four** marks. **(4×4=16)**

15. Prove that the set $\mathbb{R}$ of real numbers is not countable.

16. Let $X = (x_n)$ and $Z = (z_n)$ be sequences of real numbers that converges to x and z, respectively, where z_n and z are nonzero real numbers. Then show that X/Z converges to x/z.

17. Let A be an infinite subset of $\mathbb{R}$ that is bounded above and let $u = \sup A$. Show that there exists an increasing sequence (x_n) with $x_n \in A$ for all $n \in \mathbb{N}$ such that $u = \lim(x_n)$.

18. Show that every contractive sequence is a Cauchy sequence and therefore is convergent.

19. State and prove Abel's test for the convergence of the product of two series.

20. Let $I = [a, b]$ be a closed, bounded interval and let $f : I \to \mathbb{R}$ be continuous on I. If $k \in \mathbb{R}$ is any number satisfying $\inf f(I) \leq k \leq \sup f(I)$, then prove that there exists a number $c \in I$ such that $f(c) = k$.

SECTION - D

Answer any **Two** questions Each question carries **six** marks. **(2×6=12)**

21. a) If $S = \left\{ \frac{1}{n} : n \in S \right\}$ then prove that inf $S = 0$. **(2)**

b) Prove that there exists a positive number x such that $x^2 = 2$. **(4)**

22. a) Let $X = (x_n : n \in \mathbb{N})$ be a sequence of real numbers and let $m \in \mathbb{N}$. Then prove that the m-tail converges if X converges. **(2)**

b) Let $a > 0$. Construct a sequence s_n of real numbers that converges to $\sqrt{a}$. **(4)**

23. a) Let $X = (x_n)$ be a sequence in $\mathbb{R}$ and suppose that the limit $r = \lim |x_n|^{\frac{1}{n}}$ exists in $\mathbb{R}$. Then prove that $\sum x_n$ is absolutely convergent when $r < 1$ and is divergent when $r > 1$. **(3)**

b) Show that the absolute value function $f(x) = |x|$ is continuous at every point $c \in \mathbb{R}$. **(3)**

24. a) What do you mean by saying that a function is bounded on a subset of $\mathbb{R}$. Give an example of a bounded set. **(2)**

b) Show that every polynomial of odd degree with real coefficients has at leat one real root. **(4)**
