0153668 K19U 2259

Reg. No. : .....

Name : .....

V Semester B.Sc. Degree (CBCSS-Reg)
Examination, November - 2019
(2017 Admn. Only)
CORE COURSE IN MATHEMATICS
5B09 MAT- GRAPH THEORY

Time : 3 Hours Max. Marks : 48

SECTION - A

Answer **all** the questions. Each question carries **1** mark. **(4×1=4)**

1. What is the minimum number of edges of a simple connected graph on n vertices?
2. Define an Eulerian graph.
3. Draw $K_{2,3}$ and write the number of edges in that graph.
4. State Redei's theorem.

SECTION - B

Answer any **Eight** Questions. Each Question carries **2** marks **(8×2=16)**

5. Define Clique of a graph. Give one example.
6. Define a graphical sequence. Can (1, 1, 1, 2, 2) be a graphical sequence? Give reason.
7. Give example of a 3-regular graph. Is it possible to draw a 3-regular graph on 5 vertices ? Give reason.
8. Define Complement of a graph.
   Give example of a simple graph and its complement where both are connected.
9. If the edge e = $x - y$ of a connected graph G is a cut edge, then prove that e does not belong to any cycle of G.
10. In a tree, prove that any two distinct vertices are connected by a unique path.
11. Prove that a connected graph with n vertices and $n - 1$ edges is a tree.
12. Define a maximal independent set of vertices of a graph.

P.T.O.

13. If a graph is Eulerian, prove that the degree of each vertex of G is an even positive integer.
14. Give example of a graph that is Hamiltonian.

SECTION - C

Answer any **Four** Questions. Each Question carries **4** marks. **(4×4=16)**

15. Prove that in any group of n persons, $(n \geq 2)$ there are at least two with the same number of friends.
16. if a graph is bipartite, then prove that it contains no odd cycles.
17. Define the **union, intersection** and **join** of two graphs. Give one example for each.
18. A connected graph G with at least two vertices contains at least two vertices that are not cut vertices.
19. Prove that every connected graph contains a spanning tree.
20. In a graph G if each edge e belongs to an odd number of cycles of G, then prove that G is Eulerian.

SECTION - D

Answer any **Two** Questions. Each Question carries **6** marks. **(2×6=2)**

21. In a simple graph G with n vertices, if $\delta \geq \frac{n-1}{2}$, then prove that G is connected. What happens if the condition 'simple' is dropped ?
22. Prove that a vertex v of a connected graph G with at least three vertices is a cut vertex of G iff there exist vertices u and w of G distinct from v such that v is in every $u - w$ path in G.
23. If G is a connected labelled graph, what is the meaning of $\tau(G)$?
    If e is not a loop of a connected graph G, then prove that $\tau(G) = \tau(G-e) + \tau(Goe)$
24. For a non-trivial connected graph G, prove that the degree of each vertex of G is an even positive integer if and only if G is an edge-disjoint union of cycles.