K18U 1478

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – Reg./Sup./Imp.) Examination, November 2018
(2014 Admn. Onwards)
CORE COURSE IN MATHEMATICS
5B08 MAT – Vector Calculus

Time : 3 Hours Max. Marks: 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each** : **(4×1=4)**

1. State the Orthogonal Gradient Theorem.
2. Define Laplacian of f(x, y, z).
3. Define the circular density of vector field F = Mi + Nj at the point (x, y).
4. State Stokes theorem.

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carries **2** marks **each** : **(8×2=16)**

5. Find the angle between the planes x + y = 1 and 2x + y – 2z = 2.
6. Let $r(t) = (3t + 1)\, i + (\sqrt{3}t)\, j + t^2 k$ find the angle between the velocity and acceleration vectors at time t = 0.
7. Find the equation of the tangent plane to the surface $x^2 + y^2 - z^2 = 18$ at (3, 5, – 4).
8. Find the critical point of the function $f(x, y) = x^2 + xy + y^2 + 3x - 3y + 4$.
9. Let v = xyz (xi + yj + zk). Find curl v.
10. Let $v = e^x i + ye^{-x} j + 2z \sinh x\, k$. Prove that div $v = 2e^x$.
11. Evaluate $\int_C (x+y)\, ds$ where C is the straight-line segment x = t, y = (1 – t), z = 0 from (0, 1, 0) to (1, 0, 0).
12. Prove that the field $F = e^x \cos y i - e^x \sin y j + zk$ is conservative.

P.T.O.

13. Find a Parametrization of the surface $x^2 + y^2 + z^2 = a$.

14. Show that the flux of the position vector field F = xi + yj + zk outward through a smooth closed surface S is three times the volume of the region enclosed by the surface.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carries **4** marks **each**. **(4×4=16)**

15. Find the plane determined by the intersecting lines

$L_1 : x = t, y = -t + 2, z = t + 1, \quad -\infty < t < \infty$

$L_2 : x = 2s + 2, y = s + 3, z = 5s + 6, \quad -\infty < s < \infty$

16. Let r(t) = (6 sin 2t) i + (6 cos 2t) j + 5tk. Find T, N and k.

17. Find Quadratic and Cubic approximation of $f(x, y) = \cos(x^2 + y^2)$.

18. Prove that curl (fv) = f curl v + $\nabla f \times v$.

19. Find the potential function f(x, y, z) for the field

F = (y sin z) i + (x sin z) j + (xy cos z)k.

20. Find the surface area of a sphere of radius a.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carries **6** marks **each**. **(2×6=12)**

21. Let r(t) = (cos t) i + (sin t) j – k. Find equations for osculating, normal and rectifying planes at $t = \frac{\pi}{4}$.

22. Using the method of Lagrange multipliers find the greatest and smallest values of the function f(x, y) = xy takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$.

23. Verify both forms of the Green's theorem for the field F(x, y) = – yi + xj and the region bounded by the circle $x^2 + y^2 = a^2$ that is r(t) = a cos t i + a sin t j, $0 \le t \le 2\pi$.

24. Verify Stokes Theorem for the vector field

F(x, y, z) = (z – y) i + (z + x) j – (x + y) k; σ is the portion of the paraboloid $z = 9 - x^2 - y^2$ above the xy-plane.
