K18U 1475

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS-Reg./Sup./Imp.) Examination, November 2018
(2014 Admn. Onwards)
CORE COURSE IN MATHEMATICS
5B05 MAT : Real Analysis

Time : 3 Hours Max. Marks : 48

SECTION – A

All the **first 4** questions are **compulsory**. **Each** question carries **1** mark. **(4×1=4)**

1. Define the absolute value of a real number.
2. What is meant by a monotone sequence of real numbers ?
3. State the ratio test for convergence of series.
4. State the Bolzano's intermediate value theorem.

SECTION – B

Answer **any 8** questions. **Each** question carries **2** marks. **(8×2=16)**

5. Prove that if ab = 0, then either a = 0 or b = 0. State the algebraic properties of $\mathbb{R}$ used in the proof.
6. Prove that $|x - a| < \epsilon$ if and only if $a - \epsilon < x < a + \epsilon$.
7. If A and B are bounded subsets of $\mathbb{R}$, prove that $A \cup B$ is also bounded.
8. Prove that the sequence $(-1)^n$ is divergent.
9. If (x_n) is a sequence of non negative real numbers, prove that $\lim x_n \geq 0$.
10. Assuming that the series $\sum \frac{1}{n^2}$ converges, prove that $\sum \frac{1}{n^2 + n}$ converges.
11. Prove that if $\sum x_n$ is absolutely convergent, then it is convergent.

P.T.O.

12. State and prove Abel's test for convergence of series.

13. Prove that f (x) = |x| is continuous everywhere in $\mathbb{R}$.

14. Is $g(x) = \sqrt{x}$ uniformly continuous on [0, 2] ? Justify.

SECTION – C

Answer **any 4** questions. **Each** question carries **4** marks. **(4×4=16)**

15. State and prove the Archimedean property.

16. Let A, B be bounded nonempty subsets of $\mathbb{R}$. Let $A + B = \{a + b : a \in A, b \in B\}$. Prove that
    a) sup (A + B) = sup A + sup B and
    b) inf(A+B) = inf A + inf B.

17. Prove that every contractive sequence is a Cauchy sequence.

18. Prove that the p-series $\sum \frac{1}{n^p}$ converges when $p < 1$.

19. State and prove the alternating series test.

20. Prove that a continuous function on a closed and bounded interval is bounded.

SECTION – D

Answer **any 2** questions. **Each** question carries **six** marks. **(2×6=12)**

21. a) State and prove the nested interval property.
    b) If $I_n = [a, b], n \in \mathbb{N}$ is a nested sequence of closed and bounded invervals such that infimum of the lengths $b_n - a_n$ is 0, prove that the number ξ contained in I_n for all $n \in \mathbb{N}$ is unique.

22. a) Prove that a convergent sequence is bounded.
    b) State and prove the monotone convergence theorem.

23. a) If a series converges, prove that any series obtained by grouping its terms also converges to the same limit.
    b) State and prove the rearrangement of series theorem.

24. State and prove the location of roots theorem.

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