K18U 1479

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS-Reg./Sup./Imp.) Examination, November 2018
(2014 Admn. Onwards)
Core Course in Mathematics
5B09 MAT : GRAPH THEORY

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. **Each** question carry **1** mark. (4×1=4)

1. Define a graph.

2. Define a vertex cut.

3. What is the independence number of a graph G ?

4. Define a symmetric digraph.

SECTION – B

Answer **any 8** questions. **Each** question carries **2** marks. (8×2=16)

5. Define a self-complementary graph. Draw a graph which is self-complementary. Draw its complement also.

6. Prove that the sum of the degrees of the vertices of a graph is equal to twice the number of its edges.

7. Draw a 3-cycle and a 4-cycle. Also draw their sum.

8. If {x, y} is a 2-edge cut of a graph G, show that every cycle of G that contains x must also contain y.

P.T.O.

9. Prove that a vertex of G that is not a cut vertex belongs to exactly one of its blocks.

10. Prove that every connected graph contains a spanning tree.

11. Prove that a subset S of V is independent if and only if V/s is a covering of G.

12. Prove that if a nontrivial connected graph G is Eulerian, then the degree of each vertex of G is an even positive integer.

13. Draw a digraph which is disconnected while the underlying graph is connected.

14. How many orientations does a simple graph of m edges have ?

SECTION – C

Answer **any 4** questions. **Each** question carries **4** marks. **(4×4=16)**

15. Prove that in any group of n persons where $n \geq 2$ there are at least two with the same number of friends.

16. Prove that if e is not a loop of a connected graph G, then $\tau(G) = \tau(G - e) + \tau(G \circ e)$.

17. For any graph G for which $\delta > 0$, prove that $\alpha' + \beta' = n$.

18. If G is Hamilton, then prove that for every nonempty proper subset S of V, $\omega(G - S) \leq |S|$.

19. Prove that every tournament contains a directed Hamilton path.

20. a) Show that if a tournament contains a spanning directed cycle, then it contains a directed cycle of length 3.

    b) Show that every tournament of order n has at most one vertex v with $d^+(v) = n - 1$.

SECTION – D

Answer **any 2** questions. **Each** question carries **6** marks. **(2×6=12)**

21. a) Prove that the line graph of a simple graph G is a path if and only if G is a path.

    b) Show that the line graph of the star $K_{1,4}$ is the complete graph K_4.

22. a) For any loopless connected graph G, prove that $\kappa(G) \leq \lambda(G) \leq \delta(G)$.

    b) If G is a complete graph, what change happens to this inequality ?

23. a) Prove that the number of edges in a tree on n vertices is n – 1. Prove also the converse that a connected graph on n vertices and n – 1 edges is a tree.

    b) Prove that a tree with at least two vertices contains at least two pendant vertices.

24. a) Let G be a simple graph with $n \geq 3$ vertices. For every pair of nonadjacent vertices u, v of G if $d(u) + d(v) \geq n$ prove that G is Hamiltonian.

    b) Let G be a simple graph with $n \geq 3$ vertices. For every pair of nonadjacent vertices u, v of G if $d(u) + d(v) \geq n - 1$ prove that G is traceable.