



K18U 1477

Reg. No. :

Name :

V Semester B.Sc. Degree (CBCSS-Reg./Sup./Imp.)
Examination, November 2018
(2014 Admn. Onwards)

CORE COURSE IN MATHEMATICS

5B07 MAT-Differential Equations, Laplace Transform and Fourier Series

Time : 3 Hours

Max. Marks : 48

SECTION – A

All the first 4 questions are **compulsory**. They carry 1 mark each. (4×1=4)

1. Solve $y' + 3x^2y^2 = 0$.

2. Find the eigenvalues of the matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$.

3. Define even function. If $f(x)$ is an even function, show that

$$\int_{-L}^L f(x) dx = 2 \int_0^L f(x) dx.$$

4. Let $f(x)$ is a p periodic function. Show that $f(x)$ is np periodic for $n = 2, 3, \dots$

SECTION – B

Answer **any 8** questions from among the questions 5 to 14. These questions carry 2 marks each. (8×2=16)

5. State existence and uniqueness theorem for the initial value problem
 $y' = f(x, y) \quad y(x_0) = y_0$.

6. Show that e^x is an integrating factor of the differential equation
 $\sin y dx + \cos y dy = 0$ and hence solve it.

7. Define Wronskian of two functions f, g . Show that if f, g are linearly dependent then $W(f, g) = 0$.

P.T.O.



8. Find the general solution of $x^2y'' + xy' + y = 0$.
9. Solve $y'' + 2ky' + (k^2 + 4)y = 0$.
10. Solve $y'' - 3y' + 2y = 2e^x + 6e^{2x}$.
11. Find the inverse Laplace transform of $\frac{s-4}{s^2+4}$.
12. Find the Laplace transform of $\cosh at$.
13. Find the Fourier integral representation of the function $f(x) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$.
14. If $L(f)$ denote the Laplace transform of the function $f(x)$. Show that
 $L(f_1 + f_2) = L(f_1) + L(f_2)$, $L(cf) = cL(f)$.

SECTION - C

Answer **any 4** questions from among the questions **15 to 20**. These questions carry **4 marks each**. (4×4=16)

15. Find the general solution of the differential equation $y' + xy = 4x$.
16. Verify that $-\frac{1}{2}e^x \sin x$ is a solution of the differential equation $(D^2 - 4D + 4)y = e^x \cos x$ and find a general solution.
17. Find the inverse transform of the function $\ln\left(1 + \frac{\omega^2}{s^2}\right)$.
18. If $L(f)$ denote the Laplace transform of the function $f(x)$. Find f such that

$$L(f) = \frac{s}{(s+1)^2}$$
19. Find the Fourier series of the function

$$f(x) = \begin{cases} 0 & \text{if } -2 < x < -1 \\ k & \text{if } -1 < x < 1 \\ 0 & \text{if } 1 < x < 2 \end{cases}$$
20. Find the Fourier coefficients of the 2π periodic function defined by
 $f(x) = x + |x|, -\pi < x < \pi$.



SECTION – D

Answer **any 2** questions from among the questions **21 to 24**. These questions carry **6 marks each**. **(2×6=12)**

21. Find the orthogonal trajectories of the family of curves $xy = c$.

22. Solve the initial value problem

$$y'' - 3y' + 2y = 12x^2 + 6x^3 - x^4, \quad y(0) = 4, \quad y'(0) = -8.$$

23. Using Laplace transform solve the integral equation

$$y = 2t - 4 \int_0^t y(\tau)(t - \tau) d\tau.$$

24. Find the Fourier series of the following periodic function $f(x)$ of period $p = 2L$ defined by $f(x) = x + x^2, -1 < x < 1$.
