K18U 1476

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – Reg./Sup./Imp.) Examination, November 2018
(2014 Admn. Onwards)
CORE COURSE IN MATHEMATICS
5B06 MAT : Abstract Algebra

Time : 3 Hours Max. Marks : 48

SECTION – A

All the **first 4** questions are **compulsory**. They carry **1** mark **each** : **(4×1=4)**

1. How many binary operations can be defined on a set containing n elements ?

2. Find the number of elements in the set $\{\sigma \in S_5 | \sigma(2) = 5\}$.

3. State True or False : Every factor group of a finite group is finite.

4. Write all the units in $\mathbb{Z} \times \mathbb{Z}$.

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each** : **(8×2=16)**

5. Show that if G is a finite group with identity e and an even number of elements, then there is an element a ≠ e such that $a * a = e$.

6. Show that the left and right cancellation laws hold in a group.

7. Compute θ^2 and θ^{-1} where $\theta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$ is a permutation on S = {1, 2, 3, 4, 5}.

8. Is the converse of Langrange's theorem true ? Justify your answer.

9. Find all the cosets of the subgroup $4\mathbb{Z}$ of $\mathbb{Z}$.

P.T.O.

10. Let G be a group, $g \in G$ and $\phi_g : G \to G$ be the function $\phi_g(x) = gx, \forall x \in G$. Find all $g \in G$ such that ϕ_g is a homomorphism.

11. Show that the kernel of a homomorphism from a group G into a group G' is a normal subgroup of G.

12. Find the order of the factor group $\mathbb{Z}_6/<3>$.

13. Let R be a commutative ring with characteristic 4. Compute and simplify $(a+b)^4$ for $a, b \in R$.

14. Find all the units of $\mathbb{Z}_{14}$.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry 4 marks **each**. **(4×4=16)**

15. Show that a group with no proper non-trivial subgroups is cyclic.

16. Prove that a group G is abelian if and only if $(ab)^2 = a^2b^2$ for all $a, b \in G$.

17. Find the left regular representation of the group given in the following table :

	e	a	b
e	e	a	b
a	a	b	e
b	b	e	a

18. Suppose that G and G' are groups with identities e and e' respectively, $a \in G$ and $H \subset G$ is a subgroup. If $\phi : G \to G'$ is a homomorphism, then prove that

    i) $\phi(e) = e'$

    ii) $\phi(a^{-1}) = \phi(a)^{-1}$ and

    iii) $\phi[H]$ is a subgroup of G'.

19. Show that in a ring R, if $a^2 = a \ \forall a \in R$, then R is a commutative ring.

20. Find all solutions of the congruence $15x \equiv 27 \pmod{18}$.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**. **(2×6=12)**

21. Let G = < a > be a cyclic group of n elements, b = a^s and d = gcd(s, n). Show that < a^s > has n/d elements. Also show that < a^t > = < a^s > iff gcd(t, n) = gcd (s, n).

22. State and prove Cayley's theorem.

23. Let H be a subgroup of a group G. Show that the left coset multiplication (aH) (bH) = (ab) H is well defined iff H is a normal subgroup of G. Also show that G/H is a group when H is normal in G.

24. a) Show that in the ring $\mathbb{Z}_n$, the divisors of zero are those non-zero elements that are not relatively prime to n. **3**

    b) Show that every finite integral domain is a field. Deduce that $\mathbb{Z}_p$ is a field when p is prime. **(2+1)**