K17U 1698

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – Reg./Sup./Imp.) Examination, November 2017 (2014 Admn. Onwards) CORE COURSE IN MATHEMATICS 5B08 MAT : Vector Calculus

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions. **Each** question carries **one** mark.

1. Find the gradient of $f(x, y) = ln(x^2 + y^2)$ at point (1, 1).
2. Find the divergence of the vector function $[e^{2x} \cos 2y, e^{2x} \sin 2y, 5e^{2z}]$.
3. Show that the field $F = yi + (x + z)j - yk$ is not conservative.
4. Give a parametrization of the cone $z = \sqrt{x^2 + y^2}$, $0 \le z \le 1$. **(4×1=4)**

SECTION – B

Answer **any 8** questions. **Each** question carries **two** marks.

5. Find an equation for the plane through A(0, 0, 1), B(2, 0, 0) and C(0, 3, 0).
6. Find the unit tangent vector of the curve $r(t) = t^2 i + (2 \cos t)j + (2 \sin t)k$.
7. Is there a vector field v on R^3 such that curl v = [x sin y, cos y, z – xy] ? Justify.
8. Find equations for the tangent plane and normal line at the point (2, 0, 2) on the surface $2z - x^2 = 0$.
9. Find the local extreme values of the function.

   $f(x, y) = x^2 + 3xy + 3y^2 - 6x + 3y - 6$.
10. If $w = x^2 + y^2 + z^2$ and $z = x^2 + y^2$, find $\left(\frac{\partial w}{\partial z}\right)_x$.

P.T.O.

11. Find the work done by force $F = 3yi + 2xj + 4zk$ from (0, 0, 0) to (1, 1, 1) over the curved path $r(t) = ti + t^2j + t^4k$, $0 \leq t \leq 1$.

12. Find a potential function f for the field $F = y \sin zi + x \sin zj + xy \cos zk$.

13. Find the area of the surface cut from the paraboloid $x^2 + y^2 - z = 0$ by the plane $z = 2$.

14. Integrate $G(x, y, z) = x^2$ over the cone $z = \sqrt{x^2 + y^2}$, $0 \leq z \leq 1$. **(8×2=16)**

SECTION – C

Answer **any 4** questions. **Each** question carries **four** marks.

15. Find the length of the curve $r(t) = \sqrt{2}ti + \sqrt{2}tj + (1-t^2)\, k$ for $0 \leq t \leq 1$.

16. Find the curvature of the space curve, $r(t) = 3 \sin ti + 3 \cos tj + 4tk$.

17. Determine the constants a and b such that

    $v = [2xy + 3yz, x^2 + axz - 4z^2, 3xy + 2byz]$ is irrotational.

18. Find a quadratic approximation to $f(x, y) = e^x \cos y$ near the origin.

19. Using Divergence theorem find the outward flux of $F = x^2i + y^2j + z^2k$ across the boundary of the cube cut from the first octant by the planes $x = 1$, $y = 1$ and $z = 1$.

20. Use Stoke's theorem to evaluate $\int_C F \cdot dr$, if $F = (x + y)i + (2x - z)j + (y + z)\, k$ and

    C is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0) and (0, 0, 6). **(4×4=16)**

SECTION – D

Answer **any 2** questions. **Each** question carries **six** marks.

21. Find the Binormal vector and Torsion of the space curve, $r(t) = (\cos^3 t)i + (\sin^3 t)j$, $0 < t < \pi/2$

22. Find the extreme values of $f(x, y) = xy$ subject to the constraint $g(x, y) = x^2 + y^2 - 10 = 0$.

23. Using Green's theorem find the counterclockwise circulation and outward flux for the field $F = (x - y)\, i + (y - x)j$ and the square C bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$.

24. Find the center of mass of a thin hemispherical shell of radius a and constant density δ. **(2×6=12)**
