K17U 2255

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CCSS – Sup./Imp.) Examination, November 2017
(2013 and Earlier Admissions)
CORE COURSE IN MATHEMATICS
5B05 MAT : Vector Analysis

Time : 3 Hours | Max. Weightage : 30

1. Fill in the blanks :

   a) Area of the parallelogram with sides A, B is ___________

   b) Distance from a point S to a line through P parallel to V is ___________

   c) The gradient field of a differentiable function f(x, y, z) is the field of gradient vectors ∇f = ____________

   d) If u is a differentiable vector function of t of constant length, then the value of $u \cdot \frac{du}{dt}$ = ______________ **(Weightage 1)**

Answer **any six** from the following (Weightage **1 each**) :

2. Find the volume of the parallelopiped determined by $\bar{a} = -2i + 3k$, $\bar{b} = i + 2j - k$ and $\bar{c} = 7j - 4k$.

3. Find a spherical co-ordinate equation for the cone $z = \sqrt{x^2 + y^2}$.

4. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ if f (x, y) = y sin (xy).

5. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x + 2y + z^2$, $x = r/s$, $y = r^2 + \ln s$, $z = 2r$.

P.T.O.

6. Find the derivative of $f(x, y) = xe^y + \cos(xy)$ at the point (2, 0) in the direction of $A = 3i - 4j$.

7. Find the area enclosed by the lemniscate $r^2 = 4 \cos 2\theta$.

8. State stronger form of Fubini's theorem.

9. Find the curl of the vector field $F(x, y) = (x^2 - y)\, i + (xy - y^2)\, j$.

10. State Divergence theorem. **(6×1=6 Weightage)**

Answer **any seven** from the following (Weightage **2 each**) :

11. Find the centre and radius of the sphere $x^2 + y^2 + z^2 + 3x - 4z + 1 = 0$.

12. Find a vector parallel to the line of intersection of the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$.

13. Find the principal unit normal for the helix $r(t) = a \cos t i + a \sin t j + btk$, $a, b \geq 0$, $a^2 + b^2 \neq 0$.

14. Show that the function $f(x, y) = \dfrac{2x^2y}{x^4 + y^2}$ has no limit as (x, y) approaches (0, 0).

15. Find the local extreme values of $f(x, y) = xy$.

16. Find the derivative of the function $f(x, y, z) = xy + yz + zx$ at (1, –1, 2) in the direction of $3i + 6j + k$.

17. Find the slope of the tangent to the parabola at (1, 2, 5) where the plane $x = 1$ intersects the paraboloid $z = x^2 + y^2$ in a parabola.

18. Find the center of mass of a thin hemispherical shell of radius 'a' and constant density δ.

19. Verify both forms of Green's theorem for the field $F(x, y) = (x - y)\, i + xj$ and the region R bounded by the circle $r(t) = \cos t i + \sin t j$, $0 \leq t \leq 2\pi$.

20. Show that $F = (2x - 3)\, i - zj + \cos z k$ is not conservative. **(7×2=14 Weightage)**

Answer **any three** from the following (Weightage **3 each**) :

21. Find the absolute maximum and minimum values of $f(x, y) = 2 + 2x + 2y - x^2 - y^2$ on the triangular plate in the first quadrant bounded by the lines $x = 0$, $y = 0$, $y = 9 - x$.

22. Find the curvature and torsion for the helix $r(t) = a \cos ti + a \sin tj + bt\, k$, $a, b \geq 0$, $a^2 + b^2 \neq 0$.

23. Show that $F = (e^x \cos y + yz)\, i + (xy - e^x \sin y)\, j + (xy + z)\, k$ is conservative and find a potential function for it.

24. Verify the Divergence theorem for the field $F = xi + yj + zk$ over the sphere $x^2 + y^2 + z^2 = a^2$.

25. Find the centroid of the region in the first quadrant that bounded above the line $y = x$ and bounded below by the parabola $y = x^2$. **(3×3=9 Weightage)**
