K17U 1695

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS-Reg./Sup./Imp.) Examination, November 2017
(2014 Admn. Onwards)
CORE COURSE IN MATHEMATICS
5B05 MAT : Real Analysis

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions. **Each** question carries **one** mark.

1. Find the supremum of $S = \left\{ \frac{1}{2^m} - \frac{1}{3^n} : m, n \in \mathbb{N} \right\}$.

2. Give example of an unbounded sequence which is not monotonic.

3. Show that the series $\sum_{n=1}^{\infty} \cos n$ is divergent.

4. Let $f : A \to \mathbb{R}$ for $A \subset \mathbb{R}$. State the sequential criterion for continuity of f at $a \in A$.

(4×1=4)

SECTION – B

Answer **any 8** questions. **Each** question carries **two** marks.

5. Show that if $a, b \in \mathbb{R}$ and $a \neq b$, then there exist ϵ-neighborhoods U of a and V of b such that $U \cap V = \phi$.

6. If x and y are real numbers with x < y then show that there exists a rational number r such that x < r < y.

7. Let A, B be two nonempty sets of real numbers with suprema α and β respectively. Define the set AB by $AB = \{ab : a \in A, b \in B\}$. Give an example to show that AB need not have a supremum. Show also that even if AB has a supremum, this supremum need not be equal to $\alpha\beta$.

8. Prove or disprove : If a sequence of positive terms (x_n) converges and (y_n) has the property that $0 \leq y_n \leq x_n$ for all $n \in \mathbb{N}$, then (y_n) converges.

P.T.O.

9. Show that a sequence in $\mathbb{R}$ can have at most one limit.

10. If a series $\sum x_n$ is convergent, show that any series obtained from it by grouping the terms is also convergent. Is the converse true ? Justify.

11. Test for convergence the series $\sum \frac{1}{n^2 - n + 1}$.

12. If Σa_n and Σb_n are both divergent, is $\Sigma(a_n + b_n)$ necessarily divergent ? Justify.

13. Show that the sine function is continuous on $\mathbb{R}$.

14. Let I be a closed bounded interval and let $f : I \to \mathbb{R}$ be continuous on I. Show that the set f (I) is a closed bounded interval. **(8×2=16)**

SECTION – C

Answer **any 4** questions. **Each** question carries **four** marks.

15. Show that the set $\mathbb{R}$ of real numbers is uncountable. Deduce that the set $\mathbb{R}/\mathbb{Q}$ of irrational numbers is also uncountable.

16. State and prove the Archimedean property of real numbers. Deduce that $\inf : \{1/n : n \in \mathbb{N}\} = 0$.

17. Show that a bounded monotone sequence of real numbers is convergent.

18. State Raabe's test for the absolute convergence of a series. Using the same test the convergence of $\sum \frac{n}{n+1}$.

19. a) Show that every absolutely convergent series in $\mathbb{R}$ is convergent.

    b) Test for convergence the series $\sum_{n=1}^{\infty} \frac{3^n n!}{n^n}$.

20. Let I be a closed bounded interval and let $f : I \to \mathbb{R}$ be continuous on I. Show that f is uniformly continuous on I. **(4×4=16)**

SECTION – D

Answer **any 2** questions. **Each** question carries **six** marks.

21. Show that there exists a positive real number x such that $x^2 = 2$.

22. State and prove the Cauchy criterion for the convergence of a sequence of real numbers.

23. a) Let (z_n) be a decreasing sequence of strictly positive numbers with $\lim(z_n) = 0$. Show that the series $\sum(-1)^{n+1}z_n$ is convergent.

    b) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$.

24. Let $I \subseteq \mathbb{R}$ be an interval and let $f : I \to \mathbb{R}$ be monotone on I. Show that the set of points $D \subseteq I$ at which f is discontinuous is a countable set. **(2×6=12)**