K17U 1699

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS-Reg./Sup./Imp.) Examination, November 2017
(2014 Admn. Onwards)
CORE COURSE IN MATHEMATICS
5B 09 MAT : Graph Theory

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions. **Each** question carries **one** mark.

1. What is the smallest integer n such that the complete graph with n vertices has atleast 500 edges ?
2. Give two nonisomorphic simple connected graphs G and G′ such that their line graphs are isomorphic.
3. For what values of n, a complete graph on n vertices is Eulerian ?
4. Draw a simple graph which is Eulerian but not Hamiltonian. **(4×1=4)**

SECTION – B

Answer **any 8** questions. **Each** question carries **two** marks.

5. If G is simple and $\delta \geq \frac{n-1}{2}$ then show that G is connected.
6. Show that an edge e = xy is a cut edge of a connected graph G if and only if there exist vertices u and v such that e belongs to every u – v path in G.
7. Show that a graph G with atleast three vertices is 2-connected if and only if any two vertices of G lie on a common cycle.
8. Show that a simple graph is a tree if and only if any two distinct vertices are connected by a unique path.
9. Prove that every tree is a bipartite graph.
10. Show that a simple connected graph contains atleast m – n + 1 distinct cycles.

P.T.O.

11. What are central vertices of a graph G ? Give an example of a tree with two central vertices.

12. Show that a connected graph G is a tree if and only if every edge of G is a cut edge of G.

13. Show that a subset S of the vertex set V of a graph G is independent if and only if V\S is a covering of G.

14. Let D be a digraph with no directed cycle. Prove that there exists a vertex whose indegree is 0. **(8×2=16)**

SECTION – C

Answer **any 4** questions. **Each** question carries **four** marks.

15. a) Show that the line graph of a simple graph G is a path if and only if G is a path.
    b) Illustrate the join of two vertex-disjoint graphs with an example.

16. Prove that in a connected graph with atleast three vertices, any two longest paths have a vertex in common.

17. If C is any cycle of a simple block G with atleast three vertices, show that there exists a sequence of nonseparable subgraphs $C = B_0, B_1, \ldots B_r = G$ such that B_{i+1} is an edge-disjoint union of B_i and a path P_i, where the only vertices common to B_i and P_i are the end vertices of P_i, $0 \leq i \leq r-1$.

18. Show that every Eulerian graph has an odd number of cycle decompositions.

19. Let G be simple graph with $n \geq 3$ vertices. If for every pair of nonadjacent vertices u, v of G, $d(u) + d(v) \geq n$, show that G is Hamiltonian.

20. Show that every tournament contains a directed Hamilton path. **(4×4=16)**

SECTION – D

Answer **any 2** questions. **Each** question carries **six** marks.

21. Show that a graph is bipartite if and only if it contains no odd cycles.

22. Show that the connectivity and edge connectivity of a simple cubic graph are equal.

23. For any graph G for which $\delta > 0$, show that $\alpha' + \beta' = n$.

24. Show that every vertex of a diconnected tournament T with $n \geq 3$ vertices is contained in a directed k-cycle, $3 \leq k \leq n$. **(2×6=12)**

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