K17U 2340

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CCSS – Sup./Imp.) Examination, November 2017 (2013 & Earlier Admissions) CORE COURSE IN MATHEMATICS 5B08MAT : Graph Theory

Time : 3 Hours Max. Weightage : 30

*Instruction : Answer **all** questions.*

Fill in the blanks :

1. a) Number of edges of a complete graph with 6 vertices is ______________.

   b) Minimum number of vertices in a tree with at least two edges is ___________.

   c) A path with 10 vertices has ___________________ edges.

   d) Number of spanning trees with 3 vertices is _______________ **(Wt. 0.25×4= 1)**

Answer **any six** of the following :

2. Define a bridge.
3. Draw an Euler graph with 6 vertices.
4. Give an example of a matching which is perfect.
5. Define Hamiltonian graph.
6. State first theorem on Digraph theory.
7. Draw all non isomorphic simple graphs with 3 vertices.
8. Define a directed walk.
9. Define closure of a graph.
10. Give an example of a simple graph with exactly one cut vertex. **(Wt. 6×1=6)**

P.T.O.

Answer **any seven** of the following :

11. Draw a three regular simple graph.
12. Write the incidence matrix of $K_{2,2}$.
13. Let G be a graph without any loops. If for every pair of distinct vertices u and v of G there is precisely one path from u to v, then prove that G is a tree.
14. Prove that a connected graph with n vertices has at least n – 1 edges.
15. Prove that K_5 is Euler.
16. Prove that closure of a simple graph G is Hamiltonian if G is Hamiltonian.
17. Let D be a weakly connected digraph with atleast one arc. Prove that if D is Euler then od (v) = id (v) for every vertex v.
18. Define de Bruijn sequence.
19. Prove that a strongly connected tournament is Hamiltonian.
20. Is it true that : If tournament T is Hamiltonian then it is strongly connected. Give reason. **(Wt. 7×2=14)**

Answer **any three** of the following.

21. Let G be a k – regular graph, where k is an odd number. Prove that number of edges in G is a multiple of k.
22. Let T be a tree with at least two vertices and let $P = u_0u_1 \ldots u_n$ be a longest path in T. Then prove that both u_0 and u_n have degree 1.
23. Prove that a connected graph G is Euler if and only if the degree of every vertex is even.
24. Prove that a matching M of a graph G is maximum if and only if G contains an M-augmenting path.
25. Let u and v be distinct vertices of the digraph D. Prove that every directed u – v walk in D contains a directed u – v path. **(Wt. 3×3=9)**

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