Reg. No. : ......

V Semester B.Sc. Degree (CBCSS – Reg./Sup./Imp.) Examination, November 2017 (2014 Admn. Onwards) CORE COURSE IN MATHEMATICS

5B07 MAT : Differential Equations, Laplace Transform and Fourier Series

Time: 3 Hours Max. Marks: 48

## SECTION-A

Answer all the questions. Each question carries one mark.

1. Solve:  $y' = -\sin \pi x$ .

2. When do you say a first order ODE is linear?

3. Find the Wronskian of  $y_1$  and  $y_2$  where  $y_1(x) = \cos \omega x$  and  $y_2(x) = \sin \omega x$ .

4. Solve:  $y'' + \omega^2 y = 0$ . (4×1=4)

## SECTION-B

Answer any 8 questions. Each question carries two marks.

- 5. Test for exactness and solve,  $e^{x}(\cos y dx \sin y dy) = 0$ .
- 6. Find the orthogonal trajectory of  $x^2 y^2 = c$ .
- 7. Solve the initial value problem y'' + 2y' + y = 0, y(0) = 4, y'(0) = -6.
- 8. Find the general solution to  $x^2y'' + 1.5xy' 0.5y = 0$ .
- 9. Solve: y'' + 2y' + 5y = 0.
- 10. Find an ODE for which the functions,  $e^{-2x}\cos\omega x$  and  $e^{-2x}\sin\omega x$  are solutions.
- 11. Find the Laplace transform of sint cost.



- 12. Find the Inverse Laplace Transform of  $\frac{18s-12}{9s^2-1}$ .
- 13. Find the Fourier series of the function  $f(x) = x + \pi$  if  $-\pi < x < \pi$  and  $f(x + 2\pi) = f(x)$ .

14. Find the Fourier series of the function 
$$f(x) = \begin{cases} -1 & \text{if } -2 < x < 0 \\ 1 & \text{if } 0 < x < 2 \end{cases}$$
 (8×2=16)

## SECTION - C

Answer any 4 questions. Each question carries four marks.

- 15. Solve the initial value problem,  $y' + y \tan x = \sin 2x$ , y(0) = 1.
- 16. Solve the nonhomogeneous ODE,  $y'' 4y' + 4y = x^2e^x$  by variation of parameters.
- 17. Using the convolution theorem,  $y'' + y = \sin t$ , y(0) = 0, y'(0) = 0.
- 18. Find the inverse transform of  $\frac{3s+1}{(s-1)(s^2+1)}$
- 19. Find the Fourier series of the function  $f(x) = x^2$ , -1 < x < 1. Deduce that  $1 \frac{1}{4} + \frac{1}{9} \frac{1}{16} + \dots = \frac{\pi^2}{12}.$
- 20. Find the Fourier integral representation of the function  $f(x) = \begin{cases} 1 & \text{if } 0 < x < a \\ 0 & \text{if } x > a \end{cases}$  (4×4=16)

## SECTION - D

Answer any 2 questions. Each question carries six marks.

- 21. Find an integrating factor and solve :  $(x^4 + y^2) dx xy dy = 0$ , y(2) = 1.
- 22. Find a general solution to  $y'' + 9y = \cos x + \frac{1}{3} \cos 3x$ .
- 23. Using Laplace transforms solve,  $y'' + 2y' + y = e^{-t}$ , y(0) = 0, y'(0) = 1.
- 24. Find the a) Fourier cosine series and b) Fourier sine series for the function f defined by f(x) = 2 x, 0 < x < 2. (2x6=12)