K17U 2258

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CCSS-Sup./Imp.) Examination, November 2017
(2013 and Earlier Admissions)
CORE COURSE IN MATHEMATICS
5B09 MAT : Differential Equations and Numerical Analysis

Time : 3 Hours Max. Weightage : 30

1. a) State whether the differential equation $t^2 \frac{d^2y}{dt^2} + t\frac{dy}{dt} + 2y = \sin t$ is linear or not.

   b) If $\lambda = \alpha \pm ij3$ are the roots of the characteristic equation of $ay'' + by' + c = 0$, then write its general solution.

   c) Write the heat conduction equation.

   d) State Newton's forward difference interpolation formula. **(Weightage 1)**

Answer **any six** from the following. Weightage **1 each**.

2. Verify that the functions $y_1(t) = e^t$ and $y_2(t) = \cosh t$ are solutions of the differential equation $y'' - y = 0$.

3. Find the values of r for which the differential equation $t^2y'' + 4ty' + 2y = 0$ has a solution of the form $y = t^r$, $t > 0$.

4. Determine whether the differential equation $(2x + 3) + (2y - 2)\ y' = 0$ is exact or not.

5. Find the general solution of $y'' + 5y' + 6y = 0$.

6. Find the Wronskian of $y_1 = e^{-2t}$ and $y_2 = e^{-3t}$.

7. Solve : $y'' + 4y' + 4y = 0$.

8. Find a solution of $x^3 + x - 1 = 0$ by iteration.

P.T.O.

9. Find by Taylor series method the value of y at x = 0.1 from $\frac{dy}{dx} = x - y^2$, $y(0) = 1$ correct to 2 decimal places.

10. Explain Picard's method of successive approximation for solving a first order differential equation. **(Weightage : 6×1=6)**

Answer **any 7** from the following. Weightage **2 each**.

11. Solve the initial value problem $-ty' + 2y = 4t^2$, $y(1) = 2$.

12. Show that the equation $\frac{dy}{dx} = x^2 / (1 - y^2)$ is separable and find an equation for its integral curves.

13. Solve : $t^2y^1 + 2ty - y^3 = 0$, $t > 0$.

14. Find an integrating factor for the equation $(3xy + y^2) + (x^2 + xy)\, y' = 0$ and then solve it.

15. Find a particular solution $y'' - 3y' - 4y = 2 \sin t$.

16. If $y_1(t) = t^{-1}$ is a solution of $2t^2y'' + 3ty' - y = 0$, $t > 0$, find a fundamental set of solution.

17. Find the temperature u(x, t) at any time in a metal rod 50 cm long, insulated on the sides, which initially has a uniform temperature of 20°C throughout and whose ends are maintained at 0°C for all t > 0.

18. Use Gauss elimination to solve the system $2x + y + z = 10$, $3x + 2y + 3z = 18$, $x + 4y + 9z = 16$.

19. Using trapezoidal rule, evaluate $\int_0^1 \frac{1}{1+x}\, dx$ by dividing the interval into 4 subintervals.

20. Using Simpson's rule with h = 1, evaluate the integral $I = \int_3^7 x^2 \log x \, dx$.

(Weightage : 7×2=14)

Answer **any three** from the following. (Weightage **3 each**).

21. Solve the equation $\frac{dy}{dx} = \frac{4x - x^3}{4 + y^3}$ and draw graphs of several integral curves.

    Also, find the solution passing through the point (0, 1) and determine its interval of validity.

22. Solve the initial value problem $y' = y^{1/3}, y(0) = 1$.

23. Using the method of variation of parameters, solve $y'' + 4y = 3 \operatorname{cosec} t$.

24. Given that the values :

x :	1	3	5	7
y(x) :	24	120	336	720

    Find y(8) using Newton's forward interpolation formula.

25. Using Runge-Kutta method of fourth order, compute y(0.1) and y(0.2) correct to 4 decimal places from $\frac{dy}{dx} = y - x, y(0) = 2$. **(Weightage : 3x3=9)**

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