K17U 1696

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – Reg./Sup./Imp.) Examination, November 2017 (2014 Admn. Onwards) CORE COURSE IN MATHEMATICS 5B06 MAT : Abstract Algebra

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions. **Each** question carries **one** mark.

1. Find the number of elements in the cyclic subgroup of $\mathbb{Z}_{30}$ generated by 25.
2. What are the orbits of the identity permutation σ of a set A ?
3. How many homomorphisms are there of $\mathbb{Z}$ onto $\mathbb{Z}$?
4. What are the units in $\mathbb{Z} \times \mathbb{Z}$? **(4×1=4)**

SECTION – B

Answer **any 8** questions. **Each** question carries **two** marks.

5. Show that every permutation of a finite set is a product of disjoint cycles.
6. Let H be a subgroup of a finite group G. Show that the order of H is a divisor of the order of G.
7. Show that if σ is a cycle of odd length, then σ^2 is a cycle.
8. Find all cosets of the subgroup $\langle 2 \rangle$ of $\mathbb{Z}_{12}$.
9. Express the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 2 & 6 & 3 & 7 & 4 & 5 & 1 \end{pmatrix}$ as a product of disjoint cycles and then as a product of transpositions.

P.T.O.

10. Determine the order of the element $5 + \langle 4 \rangle$ in $\mathbb{Z}_{12}/\langle 4 \rangle$.

11. Show that any group homomorphism $\phi : G \to G'$ where $|G|$ is a prime must either be the trivial homomorphism or a one-to-one map.

12. Show that in the ring $\mathbb{Z}_n$, the divisors of 0 are precisely those nonzero elements that are not relatively prime to n.

13. Show that every finite integral domain is a field.

14. If $a \in \mathbb{Z}$ and p is a prime not dividing a, show that p divides $a^{p-1} - 1$. **(8×2=16)**

SECTION – C

Answer **any 4** questions. **Each** question carries **four** marks.

15. Show that every subgroup of a cyclic group is cyclic.

16. State and prove Lagrange's theorem. Deduce that the order of an element of a finite group divides the order of the group.

17. Let G and G′ be groups and let $\phi : G \to G'$ be a one-to-one function such that $\phi(xy) = \phi(x)\phi(y)$ for all x, y $\in$ G. Show that $\phi[G]$ is a subgroup of G′ and ϕ provides an isomorphism of G with $\phi[G]$.

18. Let $\phi : G \to H$ be a group homomorphism. Show that $\phi[G]$ is abelian if and only if for all x, y $\in$ G we have $xyx^{-1}y^{-1} \in \mathrm{Ker}(\phi)$.

19. Describe all ring homomorphisms of $\mathbb{Z}$ into $\mathbb{Z} \times \mathbb{Z}$.

20. Find all solutions of the congruence $15x \equiv 27 \pmod{18}$. **(4×4=16)**

SECTION – D

Answer **any 2** questions. **Each** question carries **six** marks.

21. If a is a generator of a finite cyclic group G of order n, show that the other generators of G are the elements of the form a^r, where r is relatively prime to n.

22. List the elements of the symmetric group S_3 on 3 letters and form the multiplication table for S_3. Find all subgroups of S_3.

23. State and prove the fundamental homomorphism theorem.

24. a) Show that the cancellation laws hold in a ring R if and only if R has no divisors of 0.

    b) Show that $a^2 - b^2 = (a + b)(a - b)$ for all a and b in a ring R if and only if R is commutative.

    c) Show that 1 and p – 1 are the only elements of the field $\mathbb{Z}_p$ that are their own multiplicative inverse. **(2×6=12)**
