K17U 2257

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CCSS – Sup./Imp.) Examination, November 2017
(2013 and Earlier Admissions)
Core Course in Mathematics
5B07 MAT : ABSTRACT ALGEBRA

Time : 3 Hours Max. Weightage : 30

1. Fill in the blanks :

   a) Example for a binary operator which is not associative in the set of all integers $\mathbb{Z}$ is

   b) Order of S_3 is

   c) Order of $\mathbb{Z}/n\mathbb{Z}$ is

   d) Example for a ring is **(Weightage : 1)**

Answer **any six** from the following (Weightage **1 each**) :

2. What do you mean by a commutative operator ? Give an example for an operator which is not commutative.

3. Define a subgroup. Give any non-trival proper subgroup of $(\mathbb{Z}_4, +_4)$.

4. Give an example for a non-cyclic group of order 4 in which all of its proper subgroups are cyclic.

5. If $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 3 & 1 \end{pmatrix}$ and $\mathcal{T} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 4 & 2 & 1 \end{pmatrix}$ are two permutations on S_5, find $\sigma\mathcal{T}$.

6. Find the orbits in the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 6 & 7 & 4 & 1 & 5 & 2 \end{pmatrix}$ in S_8.

7. Define a group homomorphism.

8. What do you mean by a factor group ?

9. What is the factor group $\frac{\mathbb{Z}}{\{0\}}$?

10. Define a ring. Give an example for a ring without unity.

11. Find all solutions of the congruence $12x \equiv 27 \pmod{18}$. **(Weightage : 6×1=6)**

Answer **any seven** from the following (Weightage **2 each**) :

12. Prove that left and right cancellation laws hold in a group.

13. State and prove division algorithm for Z.

14. Find all subgroups of $\mathbb{Z}_{18}$.

15. Find the cyclic subgroups $\langle \rho_1 \rangle$ and $\langle \mu_1 \rangle$ of S_3, the symmetric group on 3 letters.

16. Prove that every permutation of a finite set can be expressed as a product of disjoint cycles.

17. Let γ be the natural map from $\mathbb{Z}$ into $\mathbb{Z}_n$ given by $\gamma(m) = r$, where r is the reminder given by the division algorithm when m is divided by n. Show that γ is a homomorphism.

18. Prove that M is a maximal normal subgroup of a group G if and only if G/M is simple.

19. If R is a ring with additive identity 0, prove that $0a = a0 = 0$ and $a(-b) = (-a)b = -(ab)$ for every $a, b \in R$.

20. Prove that $\mathbb{Z}_p$ is a field for every prime p.

21. State and prove Little Fermat theorem. **(Weightage : 7×2=14)**

Answer **any three** from the following (Weightage **3 each**) :

22. Let $*$ be defined on $\mathbb{Q}^+$, the set of all positive rational numbers by $a * b = \frac{ab}{2}$.

    Prove that $\mathbb{Q}^+$ is an abelian group with respect to $*$.

23. Define even and odd permutations. Prove that the collection of all even permutations, A_n of $\{1, 2, 3, \ldots, n\}$ form a subgroup of the symmetric group S_n.

    Also show that $O(A_n) = \frac{n!}{2}$.

24. State and prove fundamental homomorphism theorem.

25. a) Define a commutator subgroup C of a group G.

    b) Prove that if N is a normal subgroup of a group G, then G/N is abelian if and only if $C \leq N$.

26. Prove that every field is an integral domain. What about the converse ? Justify your answer.

(Weightage : 3×3=9)