Reg. No. : ....

Name : ....

K16U 1718

V Semester B.Sc. Degree (CBCSS – 2014 Admn.-Regular) Examination, November 2016
CORE COURSE IN MATHEMATICS
5B08 MAT : Vector Calculus

Time : 3 Hours   Max. Marks : 48

SECTION – A

(Answer **all** the questions. **Each** question carries **one** mark.)

1. Find the gradient of $f(x, y) = y - x$ at point (2, 1).
2. Find the divergence of the vector function $[x^3 + y^3, 3xy^2, 3zy^2]$.
3. Show that the field $F = (2x - 3)i - zj + \cos zk$ is not conservative.
4. Give a parametrization of the sphere $x^2 + y^2 + z^2 = a^2$. **(4×1=4)**

SECTION – B

(Answer **any 8** questions. **Each** question carries **two** marks.)

5. Find the distance of the point S(1, 1, 5) to the line.
   L : $x = 1 + t,\ y = 3 - t,\ z = 2t$.
6. Find the unit tangent vector of the curve $r(t) = (2\cos t)i + (2\sin t)j + \sqrt{5}\,tk$.
7. Prove or disprove : If div v = 0 then curl v = 0.
8. Find equations for the tangent plane and normal line at the point (1, –1, 4) on the surface $z^2 - 2x^2 - 2y^2 - 12 = 0$.
9. Find the local extreme values of the function.
   $f(x, y) = x^2 + xy + y^2 + 3x - 3y + 4$.
10. If $w = x^2 + y^2 + z^2$ and $z = x^2 + y^2$, find $\left(\frac{\partial w}{\partial y}\right)_z$.
11. Find the circulation of the field $F = (x - y)i + xj$ around the circle $r(t) = \cos t\, i + \sin t\, j$, $0 \le t \le 2\pi$.

P.T.O.

12. Find a potential function f for the field $F = 2xi + 3yj + 4zk$.

13. Find the area of the region cut from the plane $x + 2y + 2z = 5$ by the cylinder whose walls are $x = y^2$ and $x = 2 - y^2$.

14. Integrate $G(x, y, z) = x$ over the parabolic cylinder $y = x^2$, $0 \leq x \leq 2$, $0 \leq z \leq 3$.

(8×2=16)

SECTION – C

(Answer **any 4** questions. **Each** question carries **four** marks.)

15. Find the length of the curve $r(t) = ti + \frac{\sqrt{6}}{2} t^2 j + t^3 k$ for $-1 \leq t \leq 1$.

16. Find the curvature of the plane curve, $r(t) = ti + (\ln \cos t)j$, $-\pi/2 < t < \pi/2$.

17. Find a quadratic approximation to $f(x, y) = xe^y$ near the origin.

18. Find the derivative of the function $f(x, y) = 2xy - 3y^2$ at (5, 5) in the direction of $4i + 3j$.

19. Find the flux of $F = 4xzi - y^2j + yzk$ outward through the surface of the cube cut from the first octant by the planes $x = 1$, $y = 1$ and $z = 1$.

20. Using Stoke's theorem calculate the circulation of the field $F = x^2i + 2xj + z^2k$ around the ellipse $4x^2 + y^2 = 4$ in the xy – plane, counterclockwise when viewed from above.

(4×4=16)

SECTION – D

(Answer **any 2** questions. **Each** question carries **six** marks.)

21. Find the Binormal vector and Torsion of the space curve, $r(t) = 3 \sin ti + 3 \cos tj + 4tk$.

22. Find the points on the ellipse $x^2 + 2y^2 = 1$ where $f(x, y) = xy$ as its extreme values.

23. Using Green's theorem find the counterclockwise circulation and outward flux for the field $F = (x^2 + 4y)i + (x + y^2)j$ and the square C bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$.

24. Find the center of mass of a thin hemispherical shell of radius α and constant density δ.

(2×6=12)