K16U 1572

Reg. No.: ....

Name: ....

V Semester B.Sc. Degree (CCSS – Supple./Imp.) Examination, November 2016
CORE COURSE IN MATHEMATICS
5B 05 MAT : Vector Analysis
(2013 & Earlier Admissions)

Time : 3 Hours Max. Weightage : 30

1. Fill in the blanks :

   a) The workdone by a force $\vec{F}$ acting through a displacement $\bar{D}$ is ________

   b) Distance from a point S to a line through P parallel to $\bar{v}$ is ________

   c) If $\bar{u}$ is a differentiable vector function of t of constant length, then the value of $\bar{u} \cdot \frac{d\bar{u}}{dt} =$ ________

   d) Vector formula for curvature is ________ **(Weightage 1)**

Answer **any six** from the following. (Weightage **1 each**)

2. Find the volume of the parallelopiped determined by $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}, \vec{b} = -2\hat{i} + 3\hat{k}$ and $\vec{c} = 7\hat{j} - 4\hat{k}$.

3. Find the spherical coordinate equation for the sphere $x^2 + y^2 + (z - 1)^2 = 1$.

4. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point (4, – 5) if $f(x, y) = x^2 + 3xy + y - 1$.

5. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x^2 + y^2$, $x = r - s$, $y = r + s$.

6. What do you mean by directional derivative of a vector field ?

P.T.O.

7. Sketch the region of integration and evaluate $\int_1^2 \int_y^{y^2} dx\, dy$.

8. Define average value of a function in space.

9. Find curl of $\vec{F} = (x^2 - y)\hat{i} + (xy - y^2)\hat{j}$.

10. State Divergence theorem. **(6×1=6 Weightage)**

Answer **any seven** from the following. **(Weightage 2 each)**

11. Find the centre and radius of the sphere $3x^2 + 3y^2 + 3z^2 + 2y - 2z = 9$.

12. Find parametric equations for the line in which the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$ intersect.

13. Find the principal unit normal for the helix $\vec{r}(t) = a\cos t\,\hat{i} + a\sin t\,\hat{j} + bt\hat{k}$,

    $a, b \geq 0$, $a^2 + b^2 \neq 0$.

14. Show that $f(x, y) = \dfrac{2xy}{x^2 + y^2}$ if $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$ is continuous at every point except the origin.

15. Find the linearization of $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$ at the point (3, 2).

16. Find the derivative of $f(x, y, z) = x^3 - xy^2 - z$ at (1, 1, 0) in the direction of the vector $2\hat{i} - 3\hat{j} + 6\hat{k}$.

17. Find the centroid of the region in the first quadrant that is bounded above the line $y = x$ and below the parabola $y = x^2$.

18. Convert the integral $\int_{-1}^{1} \int_0^{\sqrt{1-y^2}} \int_0^x (x^2 + y^2)\, dz\, dx\, dy$ into an equivalent integral in cylindrical coordinates and evaluate the result.

19. Find the flux of $\vec{F} = yz\,\hat{j} + z^2\,\hat{k}$ outward through the surface S cut from the cylinder $y^2 + z^2 = 1$, $z \geq 0$, by the planes $x = 0$ and $x = 1$.

20. Find the surface area of a sphere of radius a. **(7×2=14 Weightage)**

Answer **any three** from the following. **(Weightage 3 each)**

21. Find the maximum and minimum values of $f(x, y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$.

22. Find the volume of the region enclosed by the cylinder $x^2 + y^2 = 4$ and the plane $z = 0$ and $x + y + z = 4$.

23. Evaluate $\int_0^3 \int_0^4 \int_{x-y/2}^{x=(y/2)+1} \left(\frac{2x-y}{2} + \frac{z}{3}\right) dx\,dy\,dz$ by applying the transformation

    $u = \frac{2x-y}{2}, v = \frac{y}{2}, w = \frac{z}{3}$.

24. Find the centre of mass of a thin shell of constant density δ cut from the cone $z = \sqrt{x^2 + y^2}$ by the planes $z = 1$ and $z = 2$.

25. Use Stokes's theorem to evaluate $\int_C \vec{F} \cdot d\bar{r}$, if $\vec{F} = xz\hat{i} + xy\hat{j} + 3xz\hat{k}$ and C is the boundary of the portion of the plane $2x + y + z = 2$ in the first octant, traversed counter clockwise as viewed from above. **(3×3=9 Weightage)**
