K16U 1573

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CCSS – Supple./Imp.) Examination, November 2016
CORE COURSE IN MATHEMATICS
5B06 MAT : Real Analysis
(2013 and Earlier Admissions)

Time : 3 Hours Max. Weightage : 30

1. Fill in the blanks :

   a) The set of all $x \in \mathbb{R}$ that satisfy $|2x+3| < 7$ is ____________

   b) If $a \in \mathbb{R}$ is such that $0 \leq a < \varepsilon$ for every $\varepsilon > 0$, then a = ____________

   c) $\inf \left\{1 - \frac{(-1)^n}{n} : n \in \mathbb{N}\right\} =$ ____________

   d) The ε - neighbourhood of $a \in \mathbb{R}$ is ____________ **(Wt.= 1)**

Answer **any six** questions from the following (Weightage **one each**) :

2. If x > – 1, show that $(1+x)^n \geq 1 + nx$, for all $n \in \mathbb{N}$.

3. If $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$, prove that inf S = 0.

4. Using the definition of the limit of a sequence, prove that $\lim\left(\frac{1}{n}\right) = 0$.

5. If $X = (x_n)$, $Y = (y_n)$ are convergent sequences of real numbers and if $x_n \leq y_n$, show that $\lim(x_n) \leq \lim(y_n)$.

P.T.O.

6. If the sequence of reals $X = (x_n)$ converges to x, show that the sequence $(|x_n|)$ of absolute values converges to $|x|$.

7. Prove that a Cauchy sequence of real numbers is bounded.

8. State the integral test for the convergence of a series.

9. If $f : I \to \mathbb{R}$ is continuous on I, where $I = [a, b]$ is a closed, bounded interval and if $K \in \mathbb{R}$ is a number satisfying $\inf f(I) \leq K \leq \sup f(I)$, prove that there exists a number $c \in I$ such that $f(c) = K$.

10. If $f : A \to \mathbb{R}$, where $A \subseteq \mathbb{R}$, is uniformly continuous on A and if (x_n) is a Cauchy sequence in A, prove that $(f(x_n))$ is a Cauchy sequence in $\mathbb{R}$. **(Weightage : 6×1=6)**

Answer **any seven** questions from the following (Weightage **2 each**) :

11. If x and y are any two real numbers with $x < y$, show that there exists a rational number $r \in Q$ such that $x < r < y$.

12. Prove that the set $\mathbb{R}$ of real numbers is not countable.

13. If $0 < c < 1$, prove that $\lim \left(c^{1/n}\right) = 1$.

14. If $X = (x_n)$ and $Y = (y_n)$ are sequences of real numbers that converge to x and y respectively, show that X .Y converges to x.y.

15. Prove that a bounded sequence of real numbers has a convergent sub-sequence.

16. Show that the series $\sum_{n=1}^{\infty} 1/n!$ is convergent.

17. If $X = (x_n)$ is a convergent monotone sequence and if the series $\sum y_n$ is convergent, prove that the series $\sum x_n y_n$ is convergent.

18. If $I = [a, b]$ is a closed, bounded interval and if $f : I \to \mathbb{R}$ is continuous on I, prove that the set $f(I) = \{f(x) : x \in I\}$ is a closed bounded interval.

19. If I is a closed, bounded interval and if $f : I \to \mathbb{R}$ is continuous on I, prove that f is uniformly continuous on I.

20. If $f : I \to \mathbb{R}$ is continuous on I, where I is a closed bounded interval, prove that there exists a continuous piecewise linear function $g_\varepsilon : I \to \mathbb{R}$ such that $|f(x) - g_\varepsilon(x)| < \varepsilon$ for all $x \in I$. **(Weightage : 7×2=14)**

Answer **any three** questions from the following (Weightage **3 each**) :

21. If S is a subset of $\mathbb{R}$ that contain at least two points and has the property that $[x, y] \subseteq S$ whenever $x, y \in S$ with $x < y$, prove that S is an interval.

22. If $I_n = [a_n, b_n]$, $n \in \mathbb{N}$ is a nested sequence of closed bounded intervals such that the lengths $b_n - a_n$ of I_n satisfy $\inf\{b_n - a_n : n \in \mathbb{N}\} = 0$, prove that there is a number $\xi \in \mathbb{R}$ such that $\xi \in I_n$ for all n, and also prove that ξ is unique.

23. Prove that a monotone sequence of real numbers is convergent if and only if it is bounded.

24. Prove that a function f is uniformly continuous on the interval (a, b) if and only if it can be defined at the end points 'a' and 'b' such that the extended function is continuous on [a, b].

25. State and prove the continuous inverse theorem. **(Weightage : 3×3=9)**