K16U 1719

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – 2014 Admn. – Regular)
Examination, November 2016
CORE COURSE IN MATHEMATICS
5B09 MAT : Graph Theory

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions. **Each** question carries **one** mark.

1. How many edges are there in a complete with 1000 vertices ?
2. Draw a simple graph with degree sequence (2, 2, 3, 3).
3. Give an example of a Hamiltonian graph which is not Eulerian.
4. What is an edge covering of a graph G ? **(4×1=4)**

SECTION – B

Answer **any 8** questions. **Each** question carries **two** marks.

5. If a simple graph G has at least two vertices, show that it has two vertices of the same degree.
6. Show that an edge e of a connected graph G is a cut edge of G if and only if e belongs to no cycle of G.
7. Prove or disprove : Let G be a simple connected graph with $n(G) \geq 3$. If G has a cut vertex then G has a cut edge.
8. Show that any tree with two or more vertices has at least two pendant vertices.
9. Show that every connected graph contains a spanning tree.

P.T.O.

10. Show that a vertex v of a connected graph G with at least three vertices is a cut vertex of G if and only if there exist vertices u and w of G distinct from v such that v is in every u – w path in G.

11. Show that no vertex of a simple graph can be a cut vertex of both G and G^c.

12. Show that the number of edges in a tree with n vertices is n – 1.

13. If G is Hamiltonian, then for every nonempty proper subset S of V show that $w(G - S) \leq |S|$.

14. Show that every tournament of order n has at most one vertex v with $d^+(v) = n - 1$.

(8×2=16)

SECTION – C

Answer **any 4** questions. **Each** question carries **four** marks.

15. If two simple graphs are isomorphic, show that their line graphs are also isomorphic.

16. a) Show that if G is a self-complementary graph of order n, then $n \equiv 0$ or 1 (mod 4).

    b) Draw two non-isomorphic graphs with the same number of vertices, same number of edges and an equal number of vertices with a given degree.

17. Show that a simple cubic connected graph G has a cut vertex if and only if it has a cut edge.

18. If a connected graph G is an edge-disjoint union of cycles, show that G is Eulerian.

19. a) Show that a subset S of the vertex set V of a graph G is independent if and only if V/S is a covering of G.

    b) For any graph G, prove that $\alpha + \beta = n$.

20. Show that every tournament contains a directed Hamilton path. **(4×4=16)**

SECTION – D

Answer **any 2** questions. **Each** question carries **six** marks.

21. Show that a graph is bipartite if and only if it contains no odd cycles.

22. State and prove Cayley's formula for the number of spanning trees of a labeled complete graph.

23. Let G be a simple graph with $n \geq 3$ vertices. If $d(u) + d(v) \geq n - 1$ for every pair of nonadjacent vertices u and v of G, show that G is traceable.

24. Show that every vertex of a diconnected tournament T with $n \geq 3$ vertices is contained in a directed k-cycle, $3 \leq k \leq n$. **(2×6=12)**
