



K16U 1717

Reg. No. : .....

Name : .....

V Semester B.Sc. Degree (CBCSS–2014 Admn. – Regular) Examination,  
November 2016

CORE COURSE IN MATHEMATICS

5B07 MAT : Differential Equations, Laplace Transform and Fourier  
Series

Time : 3 Hours

Max. Marks : 48

SECTION – A

Answer **all** the questions. **Each** question carries **one** mark.

1. Solve  $\frac{dy}{dx} + \frac{x}{y} = 0$ .
2. When do you say that a first-order differential equation is exact ?
3. Find the Wronskian of  $y_1$  and  $y_2$  where  $y_1(x) = \sin x$  and  $y_2(x) = \cos x$ .
4. Solve  $y'' - y = 0$ . (4×1=4)

SECTION – B

Answer **any 8** questions. **Each** question carries **two** marks.

5. Solve  $y' \tan x = 2y - 8, y\left(\frac{1}{2}\pi\right) = 0$ .
6. Find the orthogonal trajectory of  $y = c/x$ .
7. Find an ODE  $y'' + ay' + by = 0$  for the basis  $1, e^{-3x}$ .
8. Solve the initial value problem  $y'' + y' - 2y = 0, y(0) = 4, y'(0) = -5$ .

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9. Solve  $(4D^2 + 4\pi D + \pi^2)y = 0$ .
10. Find the general solution to  $4x^2y'' + 4xy' - y = 0$ .
11. If  $f(t) = t \sin \omega t$ , show that  $L(f'') = s^2L(f)$ .
12. Find the inverse Laplace transform of  $\frac{20}{(s-1)(s+4)}$ .
13. Find the Fourier series of  $f(x) = (\pi - x)/2$  in the interval  $(0, 2\pi)$ .
14. Find the Fourier series of the  $2\pi$ -periodic function  $f$  defined by  $f(x) = x^2, -\pi < x < \pi$ .

(8×2=16)

## SECTION - C

Answer **any 4** questions. **Each** question carries **four** marks.

15. Find an integrating factor and solve the initial value problem,  $(e^{x+y} + ye^y)dx + (xe^y - 1)dy = 0, y(0) = -1$ .
16. Solve the non-homogeneous ODE,  $y'' + y = \sec x$ .
17. Solve the initial value problem  $y'' - y = t; y(0) = 1, y'(0) = 1$ , using Laplace transforms.
18. Using convolution theorem, solve  $y'' + 4y = \sin 3t, y(0) = 0, y'(0) = 0$ .
19. Find the Fourier series of the  $2\pi$ -periodic function  $f$  defined by

$$f(x) = \begin{cases} -k & \text{if } -\pi < x < 0 \\ k & \text{if } 0 < x < \pi \end{cases}$$

Deduce that  $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$ .

20. Find the Fourier integral representation of the function  $f(x) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$

(4×4=16)



SECTION – D

Answer **any 2** questions. **Each** question carries **six** marks.

21. Find the general solution and also the particular solution corresponding to the

given initial condition :  $y' \cos^2 x + 3y = 1, y\left(\frac{1}{4}\pi\right) = \frac{4}{3}$ .

22. Solve the initial value problem :

$$y'' + 3y' - 2.25y = -10e^{-15x}, y(0) = 1, y'(0) = 0.$$

23. Find the inverse transform of  $\ln\left(1 + \frac{\omega^2}{s^2}\right)$ .

24. Find the (a) Fourier cosine series and (b) Fourier sine series for the function f

defined by  $f(x) = x, 0 < x < \frac{1}{2}$ .

(2×6=12)

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