K16U 1716

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – 2014 Admn. – Regular) Examination, November 2016
CORE COURSE IN MATHEMATICS
5B06 MAT : Abstract Algebra

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions. **Each** question carries **one** mark.

1. How many generators are there for the cyclic group $\mathbb{Z}$ under addition ?

2. Write the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 2 & 4 & 3 & 1 \end{pmatrix}$ as a product of disjoint cycles.

3. Find the order of the factor group $\mathbb{Z}_6/\langle 3\rangle$.

4. What are the units in $\mathbb{Z}_4$? (4×1=4)

SECTION – B

Answer **any 8** questions. **Each** question carries **two** marks.

5. Prove that every cyclic group is abelian.

6. Show that a group with no proper nontrivial subgroup is cyclic.

7. Find all orbits of the permutation $\sigma : \mathbb{Z} \to \mathbb{Z}$ where $\sigma(n) = n + 2$.

8. Show that S_n the symmetric group on n letters is nonabelian for $n \geq 3$.

9. Find the partition of the group $\mathbb{Z}_6$ into cosets of the subgroup $H = \{0, 3\}$.

10. Find Ker (ϕ) and ϕ (25) for the homomorphism $\phi : \to \mathbb{Z}_7$ such that $\phi(1) = 4$.

P.T.O.

11. Show that if a finite group G has exactly one subgroup H of a given order, then H is a normal subgroup of G.

12. Show that the rings $2\mathbb{Z}$ and $3\mathbb{Z}$ are not isomorphic.

13. Show that the cancellation laws hold in a ring R if and only if R has no divisors of 0.

14. Show that the characteristic of an integral domain must be either 0 or a prime p.

(8×2=16)

SECTION – C

Answer **any 4** questions. **Each** question carries **four** marks.

15. Let $S = \mathbb{R}\setminus\{-1\}$. Define $*$ on S by $a * b = a + b + ab$. Show that $\langle S, *\rangle$ is a group.

16. State and prove Lagrange's theorem. Deduce that every group of prime order is cyclic.

17. Show that the collection of all even permutations of $n \geq 2$ letters forms a subgroup of order n!/2 of the symmetric group S_n.

18. Let ϕ be a homomorphism of a group G into a group G'. Prove the following.

    a) If H is a subgroup of G then ϕ [H] is a subgroup of G'.

    b) If K' is a subgroup of G' then $\phi^{-1}[K']$ is a subgroup of G.

19. Describe all ring homomorphisms of $\mathbb{Z} \times \mathbb{Z}$ into $\mathbb{Z}$.

20. Show that the set G_n of nonzero elements of $\mathbb{Z}_n$ that are not 0 divisors forms a group under multiplication modulo n.

(4×4=16)

SECTION – D

Answer **any 2** questions. **Each** question carries **six** marks.

21. Let G be a cyclic group with generator a. If the order of G is infinite, show that G is isomorphic to $\langle\mathbb{Z}, +\rangle$. Further, if G has finite order n, then show that G is isomorphic to $\langle\mathbb{Z}_n, +_n\rangle$.

22. Show that every group is isomorphic to a group of permutations.

23. a) If $\phi : G \to G'$ is a group homomorphism show that Ker (ϕ) is a normal subgroup of G.

b) Show that there are no nontrivial homomorphism $\phi : \mathbb{Z}_{12} \to \mathbb{Z}_5$.

c) Determine the order of the element $26 + \langle 12 \rangle$ in $\mathbb{Z}_{60}/\langle 12 \rangle$.

24. Let R be a ring that contains at least two elements. Suppose for each nonzero $a \in R$, there exists a unique $b \in R$ such that $aba = a$.

a) Show that R has no divisors of 0.

b) Show that $bab = b$.

c) Show that R has unity.

d) Show that R is a division ring. **(2×6=12)**
