K16U 1574

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CCSS–Supple./Imp.) Examination, November 2016
CORE COURSE IN MATHEMATICS
5B07 MAT : Abstract Algebra
(2013 and Earlier Admissions)

Time : 3 Hours Max. Weightage : 30

1. Mark each of the following **true** or **false** :
   a) A binary operation on a set S may assign more than one element of S to some ordered pairs of elements of S.
   b) In every cyclic group, every element is a generator.
   c) Every group is a subgroup of itself.
   d) $\mathbb{Z}_4$ is a cyclic group. **(Wt.1)**

Answer **any six** questions from the following (Weightage **one each**) :

2. If S is the set of all real numbers of the form $a+b\sqrt{2}$, where $a, b \in \mathbb{Q}$ are not simultaneously zero, show that S is a group under usual multiplication of real numbers.
3. If G is a group with binary operation $*$, prove that $(a*b)' = b'*a'$, for all $a, b \in G$, where a' is the inverse of a.
4. Define orbit of a permutation and find the orbits of the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 6 & 7 & 4 & 1 & 5 & 2 \end{pmatrix}$
5. Prove that every permutation σ of a finite set is a product of disjoint cycles.
6. Prove that every group of prime order is cyclic.

P.T.O.

7. If H is a normal subgroup of a group G, show that the cosets of H in G forms a group under the binary operation (aH) (bH) = (ab) H.

8. Prove that a factor group of a cyclic group is cyclic.

9. Solve the equation $x^2 - 5x + 6 = 0$ in $\mathbb{Z}_{12}$.

10. Show that $\mathbb{Z}_p$ is a field, if p is a prime.

11. If R is a ring with unity and if n.1=0 for some $n \in \mathbb{Z}^+$, then show that the smallest such n is the characteristic of R. **(6×1=6)**

Answer **any seven** questions from the following (weightage **2 each**)

12. If G is a group show that $(a * b)' = a' * b'$ if and only if $a * b = b * a$, for $a, b \in G$, where a' is the inverse of a.

13. If G is a group and $a \in G$, show that $H = \{a^n / n \in \mathbb{Z}\}$ is the smallest subgroup of G that contains 'a'.

14. Find all subgroups of $\mathbb{Z}_{18}$.

15. If H is subgroup of a finite group G, then prove that order of H is a divisor of order of G. Also prove that the order of an element of a finite group divides the order of the group.

16. Obtain the group of symmetries of a square with vertices 1, 2, 3 and 4.

17. Define a homomorphism of a group G into a group G'. If $\varphi : G \rightarrow G'$ is a homomorphism of a group G onto a group G' and if G is abelian, show that G' is also abelian.

18. If H is a normal subgroup of a group G, prove that the map $\gamma : G \rightarrow G/H$ defined by $\gamma(x) = x H$, is a homomorphism with Kernel H.

19. Prove that the cancellation law hold in a ring R if and only if R has no zero divisors.

20. Show that every field is an integral domain.

21. Show that $n^{33} - n$ is divisible by 15. **(7×2=14)**

Answer **any three** questions from the following (Weightage **3 each**)

22. Prove that a subgroup of a cyclic group is cyclic.

23. If G and G′ are groups and if $\varphi : G \to G'$ is one-to-one such that $\varphi(xy) = \varphi(x)\varphi(y)$, show that $\varphi(G)$ is a subgroup of G′.

24. Prove that the collection of all even permutations of $\{1, 2, ..., n\}$ $n \geq 2$, forms a subgroup of order $n!/2$ of the symmetric group S_n.

25. Prove that a subgroup H of a group G is a normal subgroup of G if and only if gH=Hg for all $g \in G$.

26. Show that every finite integral domain is a field. **(3×3=9)**
