M 9811

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CCSS – Reg./Supple./Imp.) Examination, November 2015
CORE COURSE IN MATHEMATICS
5 B05 MAT : Vector Analysis

Time : 3 Hours   Max. Weightage : 30

1. Fill in the blanks :

   a) Two non-zero vectors $\vec{a}$ and $\vec{b}$ are orthogonal if and only if ________

   b) Vector equation of the plane through $P_0 (x_0, y_0, z_0)$ and normal to $\vec{n}$ is ________

   c) If $\vec{v}$ is the velocity vector of a particle moving along a smooth curve in space at time t, then acceleration vector is ________

   d) If k is the curvature then center of curvature ρ is ________ **(Weightage 1)**

Answer **any six** from the following **(Weightage 1 each) :**

2. Find the area of the triangle with vertices (1, –1, 0), (2, 1, –1) and (–1, 1, 2).

3. Find the equation of the cylinder $x^2 + (y-3)^2 = 9$ in cylindrical coordinates.

4. If $f(x, y) = \dfrac{2y}{y + \cos x}$, find f_x.

5. Find $\dfrac{dw}{dt}$ at $t = \dfrac{\pi}{2}$ if $w = xy$, $x = \cos t$, $y = \sin t$.

6. State Euler's theorem on homogeneous functions.

7. Find the area of the region R bounded by $y = x$ and $y = x^2$ in the first quadrant.

8. Find the average value of $f(x, y) = x \cos(xy)$ over the rectangle $0 \le x \le \pi, 0 \le y \le 1$.

9. Find the gradient field of $\phi = xyz$.

10. State Stoke's theorem. **(Weightage 6×1=6)**

P.T.O.

Answer **any seven** from the following **(Weightage 2 each) :**

11. Find the vector projection of $\vec{b} = 6\hat{i} + 3\hat{j} + 2\hat{k}$ onto $\vec{a} = \hat{i} - 2\hat{j} - 2\hat{k}$ and scalar component of $\vec{b}$ in the direction of $\vec{a}$.

12. Find the distance from the point (1, 1, 5) to the line $x = 1 + t$, $y = 3 - t$, $z = 2t$.

13. Find the unit tangent vector and principal unit normal for the circular motion $\vec{r}(t) = \cos 2t\ \hat{i} + \sin 2t\ \hat{j}$.

14. Find $\lim_{(x,y)\to(0,0)} \dfrac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$.

15. Find the linearization of $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$ at the point (3, 2).

16. Find the derivative of $f(x, y) = xe^y + \cos(xy)$ at (2, 0) in the direction of the vector $3\hat{i} - 4\hat{j}$.

17. Find the centroid of the region in the first quadrant that is bounded above the line $y = x$ and below the parabola $y = x^2$.

18. Find the area of the region that lies inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$.

19. Show that the work done by force field $\vec{F} = yz\hat{i} + xz\hat{j} + xy\hat{k}$ is independent of path. Also find the work done along any smooth curve joining the point (–1, 3, 9) to (1, 6, –4).

20. Integrate xyz over the surface of the cube cut from the first octant by the planes $x = 1$, $y = 1$ and $z = 1$.

(Weightage 7×2=14)

Answer **any three** from the following **(Weightage 3 each) :**

21. Find the greatest and smallest values that the function $f(x, y) = xy$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$.

22. Find the volume of the upper region D cut from the solid sphere $\rho \leq 1$ by the cone $\phi = \frac{\pi}{3}$.

23. Evaluate $\int_0^1 \int_0^{1-x} \sqrt{x+y}\,(y-2x)^2\,dy\,dx$.

24. Find the flux of $\vec{F} = yz\hat{i} + x\hat{j} - z^2\hat{k}$ outward through the parabolic cylinder $y = x^2, 0 \leq x \leq 1, 0 \leq z \leq 4$.

25. Verify Green's theorem in the plane for the field $\vec{F} = (x - y)\hat{i} + x\hat{j}$ and the region R bounded by the unit circle $C : \vec{r}\,(t) = \cos t\hat{i} + \sin t\hat{j}, 0 \leq t \leq 2\pi$. **(Weightage 3×3=9)**
