M 9814

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CCSS-Reg./Supple./Imp.) Examination, November 2015
CORE COURSE IN MATHEMATICS
5B08 MAT : Graph Theory

Time : 3 Hours   Max. Weightage : 30

***Instruction** : Answer **all** questions.*

Fill in the blanks :

1. a) The number of edges in a complete graph with n vertices is ____________

   b) The join of the complements of Km and Kn is the complete bipartite graph

   c) A connected graph G with n vertices has at least ________________ edges.

   d) Let G be a simple graph on vertices with $n \geq 3$, if C(G) = Kn, then G is **(W = 1)**

Answer **any six** from the following. Wt : **1 each**.

2. Let G be a simple graph with n vertices. Define the complement $\overline{G}$ of G. Give an example.

3. Draw $K_{1, 2, 2}$ and $K_{2, 2, 2}$.

4. Define the vertex connectivity of a simple graph G with an example.

5. Define Euler tour of a graph G and Hamiltonian path in a graph G.

6. Give an example of a matching in G which is maximum but not perfect.

7. Draw the de Bruijin diagram $D_{2, 3}$.

8. Prove that a tournament T is Hamiltonian if it is strongly connected.

9. Define a K-regular digraph. Give an example of a 2-regular digraph with 5 vertices.

10. Draw the join of the graphs K_2 and K_3. **(6×1=6)**

P.T.O.

Answer **any 7** of the following. Wt : **2 each**.

11. Define graph isomorphism. Give example.
12. State and prove the first theorem of graph theory.
13. Define self-complementary graph. Give an example.
14. Let u and v be distinct vertices of a tree T. Then prove that there is precisely one path from u to v.
15. Let G be a connected graph. Then G is a tree if and only if every edge of G is a bridge.
16. Prove that a graph G is connected if and only if it has a spanning tree.
17. Let G be a graph with n vertices where $n \geq 2$. Then prove that G has at least two vertices which are not cut vertices.
18. Prove that a connected graph G has an Euler tour if and only if it has at most two odd vertices.
19. Prove that a simple graph G is Hamiltonian if and only if its closure C(G) is Hamiltonian.
20. Prove that every tournament T has a directed Hamiltonian path. **(7×2=14)**

Answer **any 3** of the following. Wt : **3 each**.

21. Let G be a non-empty graph with at least two vertices. Then prove that G is bipartite if and only if it has no odd cycle.
22. Let e be an edge of the graph G and let G-e be the subgraph obtained by deleting e, then prove that $W(G) \leq W(G-e) \leq W(G)+1$.
23. Let G be a simple graph with atleast 3 vertices .Then prove that G is 2-connected if and only if for each pair of distinct vertices u and v of G, there are two internally disjoint u-v paths in G.
24. If G is a simple graph with n vertices where $n \geq 3$ and the degree $d(V) \geq n/2$ for every vertex V of G, then prove that G is Hamiltonian.
25. Let D be a weakly connected digraph with atleast one arc. Then prove that D is Euler if and only if od(V) = id(V) for every vertex V of D. **(3×3=9)**

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