M 9815

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CCSS-Reg./Supple./Imp.) Examination, November 2015
CORE COURSE IN MATHEMATICS
5B 09 MAT : Differential Equations and Numerical Analysis

Time: 3 Hours Max. Weightage : 30

1. Fill in the blanks :

   a) Characteristic equation of $ay'' + by' + cy = 0$ is ________

   b) If the roots of the characteristic equation of $ay'' + by' + cy = 0$ is real and repeated, say $\lambda = 3, 3$, then the general solution is ________

   c) Wronskian of e^{-2t} and e^{-3t} is ________

   d) Two functions f(t) and g(t) are said to be linearly dependent if ________

   (Weightage 1)

Answer **any six** from the following : (Weightage **1 each**)

2. Determine the order of the equation $u_{xx} + u_{yy} + uu_x + uu_y + u = 0$. Also state whether the equation is linear or non-linear.

3. Solve $\frac{dy}{dt} = ay - b,\ y(0) = y_0$.

4. Find the general solution of $y'' + 9y = 0$.

5. Find the Wronskian of the vectors $x^{(1)}(t) = \begin{pmatrix} e^t \\ e^t \end{pmatrix}$ and $x^{(2)}(t) = \begin{pmatrix} t^2 \\ 2t \end{pmatrix}$.

6. Solve the boundary value problem $y'' + y = 0,\ y(0) = 1,\ y(\pi) = a$.

7. Explain one dimensional heat equation.

P.T.O.

8. Using Newton-Raphson method, find the positive solution of $2\sin x = x$.

9. What do you mean by backward differences ? State Newton's backward interpolation formula.

10. Apply Euler's method to solve the initial value problem $y' = x + y, y(0) = 0$ to find $y(0.1)$ and $y(0.2)$. Take $h = 0.1$. **(Weightage : 6×1=6)**

Answer **any seven** from the following : (Weightage **2 each**)

11. Determine the value of r for which the differential equation $t^2y'' - 4ty' + 4y = 0$ has solution of the form $y = t^r$, $r > 0$.

12. Solve $\frac{dy}{dt} + \frac{1}{2}y = 2 + t$.

13. Given that $y_1(t) = t^{-1}$ is a solution of $2t^2y'' + 3ty' - y = 0, t > 0$. Find a second linearly independent solution.

14. Find the particular integral of $y'' - 3y' - 4y = -8e^t \cos 2t$.

15. Find the solution of the initial value problem $y'' + 4y = 3\sin 2t, y(0) = 2, y'(0) = -1$.

16. Using the method of separation of variables, solve Laplace's equation.

17. Consider a elastic string of length 30 cm that satisfies the wave equation $4u_{xx} = u_{tt}$, $0 < t < 30$, $t > 0$. Assume that the ends of the strings are fixed and the string is set in motion with no initial velocity from the initial position.

$$u(x, 0) = \begin{cases} x/10 & 0 \le x \le 10 \\ (30 - x)/20 & 10 < x \le 30 \end{cases}$$

Find the displacement u(x.t) of the string.

18. Using Gauss elimination method, solve the equations $x + 2y - z = 3$; $3x - y + 2z = 1$; $2x - 2y + 3z = 2$ and $x - y + z = -1$.

19. Using Simpson's rule evaluate $\int_0^6 \frac{dx}{1+x^2}$ by dividing the interval into 10 sub-intervals.

20. Using Picard's process of successive approximation, obtain the value of y(0.1) from the equation $\frac{dy}{dx} = x - y^2$, $y(0) = 1$. **(Weightage : 7×2=14)**

Answer **any three** from the following : (Weightage **3 each**)

21. Solve the initial value problem $\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2(y-1)}$, $y(0) = -1$ and determine the interval in which the solution exist.

22. Find an integrating factor for the equation and solve $(3xy + y^2) + (x^2 + xy)\, y' = 0$.

23. Using method of variation of parameters, solve $y'' + 2y' + y = 3e^{-t}$.

24. Given that the values :

x :	5	7	11	13	17
f(x) :	150	392	1452	2366	5202

Evaluate f(9) using Newton's divided difference formula.

25. Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = x + y$ with $y = 1$, where $x = 0$ at $x = 0.2$ and $x = 0.4$ **(Weightage : 3×3=9)**

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