M 4439

Reg. No. : ....

Name : ....

V Semester B.A./B.Sc./B.Com./B.B.A./B.B.A. T.T.M./B.B.M./B.C.A./B.S.W./
B.A. Afsal-Ul-Ulama Degree (CCSS – Reg./Sup./Imp.)
Examination, November 2013

CORE COURSE IN MATHEMATICS
5 B05 MAT : Vector Analysis

Time : 3 Hours Weightage : 30

1. Fill in the blanks :

   a) The angle between two non zero vectors $\vec{a}$ and $\vec{b}$ is ______________

   b) If $f(x, y) = x^2 + 3xy + y - 1$, then $\dfrac{\partial f}{\partial x}$ at $(4, -5)$ is ______________

   c) Spherical form of volume element dV is ______________

   d) If $\vec{F}$ is a conservative field in a region D, then the value of $\int \vec{F} \cdot d\vec{r}$ around every closed loop in D is ______________ **(Weightage 1)**

Answer **any six** from the following. **(Weightage 1 each)**

2. If $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = -4\hat{i} + 3\hat{j} + \hat{k}$, find $\vec{a} \times \vec{b}$.

3. Find the equation for the circular cylinder $4x^2 + 4y^2 = 9$ in cylindrical coordinates.

4. If $w = x \sin y + y \sin x + xy$, verify that $w_{xy} = w_{yx}$.

5. Using chain rule, find the derivative of $w = xy$ with respect to t along the path $x = \cos t$, $y = \sin t$.

6. Find the plane tangent to the surface $z = x \cos y - ye^x$ at $(0, 0, 0)$.

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7. Find the area enclosed by the lemniscate $r^2 = 4\cos 2\theta$.

8. Find the average height of the paraboloid $z = x^2 + y^2$ over the square $0 \le x \le 2$, $0 \le y \le 2$.

9. Find the gradient field of $\phi = ln\sqrt{x^2+y^2+z^2}$.

10. State Stoke's theorem. **(Weightage 6×1=6)**

Answer **any seven** from the following. **(Weightage 2 each)**

11. Find the centre and radius of the sphere $x^2 + y^2 + z^2 + 3x - 4z + 1 = 0$.

12. Find the distance from (1, 1, 3) to the plane $3x + 2y + 6z = 6$.

13. Find the torsion of the helix $\vec{r}(t) = a\cos t\,\hat{i} + a\sin t\,\hat{j} + bt\,\hat{k}$, $a, b \ge 0$, $a^2 + b^2 \ne 0$.

14. Find the derivative of $f(x, y, z) = x^3 - xy^2 - z$ at P(1, 1, 0) in the direction of $2\hat{i} - 3\hat{j} + 6\hat{k}$.

15. Find the linearization of $f(x, y, z) = x^2 - xy + 3\sin z$ at the point (2, 1, 0).

16. The surfaces $f(x, y, z) = x^2 + y^2 - 2 = 0$ and $g(x, y, z) = x + z - 4 = 0$ meet in an ellipse E. Find the parametric equations for the line tangent to E at the point (1, 1, 3).

17. Change the integral $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} dy\,dx$ into an equivalent polar integral and hence evaluate it.

18. Find the average value of $x + y - z$ over the rectangular solid in the first octant bounded by the coordinate planes and the planes $x = 1$, $y = 1$ and $z = 2$.

19. Find the work done by $\vec{F} = (y - x^2)\,\hat{i} + (z - y^2)\,\hat{j} + (x - z^2)\,\hat{k}$ over the curve $\vec{r}(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}, 0 \le t \le 1$, from (0, 0 , 0) to (1, 1, 1).

20. Find the surface area of the cone $z = \sqrt{x^2 + y^2}$, $0 \le z \le 1$. **(Weightage 7×2=14)**

Answer **any three** from the following. **(Weightage 3 each)**

21. Find the maximum and minimum values of $f(x, y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$.

22. Find the volume of the region D enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

23. Evaluate : $\int_0^4 \int_{x=y/2}^{x=(y/2)+1} \frac{2x-y}{2}\, dx\, dy$ by applying the transformation

$u = \frac{2x-y}{2}, v = \frac{y}{2}$.

24. Find the flux of $\vec{F} = yz\hat{i} + x\hat{j} - z^2\hat{k}$ outward through the parabolic cylinder $y = x^2, 0 \le x \le 1, 0 \le z \le 4$.

25. Verify Green's theorem in the plane for $\int_C (xy\, dx + x^2 dy)$ where C is the curve enclosing the region bounded by the parabola $y = x^2$ and the line $y = x$.

(Weightage 3×3=9)

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