M 4440

Reg. No. : ....

Name : ....

V Semester B.A./B.Sc./B.Com./B.B.A./B.B.A. T.T.M./B.B.M./B.C.A./B.S.W. B.A. Afsal ul Ulama Degree (CCSS-Reg./Supple./Improv.) Examination, November 2013

Core Course in Mathematics

5B06 MAT – REAL ANALYSIS

Time : 3 Hours Max. Weightage : 30

1. Fill in the blanks. **(Weightage 1)**

   a) If $a < 0$, then $|a| =$ ______________ .

   b) If $In = \left[0, \frac{1}{n}\right]$ for $n \in \mathbb{N}$, then $\bigcap_{n=1}^{\infty} I_n =$ ______________

   c) $\text{Inf}\left\{1 - \frac{(-1)n}{n} = n \in \mathbb{N}\right\} =$ ______________

   d) $\text{Lim}\,(2n/_{2n+2}) =$ ______________

Answer **any six** from the following. Weight **1 each**.

2. If $S = \{x : x \text{ is rational}, x^2 + 4 < 6\}$. Find the least upper bound and greatest lower bound of S.

3. If $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$, show that inf $S = 0$.

4. Prove that a sequence in $\mathbb{R}$ can have at most one limit.

5. Show that the sequence (n) is divergent.

6. If a sequence $X = (x_n)$ of real numbers converges to a real number x, then show that any subsequence $X^1 = (x_{n_k})$ of X converges to x.

7. State the root test and Raabe's test.

8. If $X = (x_n)$ is a sequence in $\mathbb{R}$ and if $r = \lim |x_n|^{1/n}$ exists in $\mathbb{R}$, then show that $\sum x_n$ is absolutely convergent when $r < 1$ and divergent when $r > 1$.

9. Show that the cosine function is continuous on $\mathbb{R}$.

10. If $f : I \to \mathbb{R}$, $I \subseteq \mathbb{R}$, is increasing on I and $c \in I$, then show that f is continuous at 'c' if and only if the jump of f at c, $j_f(c) = 0$.

Answer **any seven** from the following. Weight **2 each**.

11. If a and b are rationals, prove that $a + b\sqrt{2}$ is irrational.

12. Prove that the unit interval [0, 1] is not countable.

13. If $X = (x_n)$ is a sequence of real numbers that converges to x and if $x_n \geq 0$ for all n, then prove that $(\sqrt{x_n})$ of positive square roots converges and $\lim(\sqrt{x_n}) = \sqrt{x}$.

14. State and prove the Bolzano Weierstrass theorem.

15. Define a Cauchy sequence and show that $\left(\frac{1}{n}\right)$ is a Cauchy sequence.

16. Show that the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ is divergent.

17. State the Dirichlet test for infinite series and using this prove that $\sum_{n=1}^{\infty} \frac{1}{n} \sin nx$, $x \neq 2k\pi, k \in \mathbb{N}$, is convergent.

18. If I is a closed bounded interval and $f : I \to \mathbb{R}$ is continuous on I, show that f is uniformly continuous on I.

19. If $I = [a, b]$ is a closed bounded interval and $f : I \rightarrow \mathbb{R}$, is continuous on I, then show that f is bounded on I.

20. Let $f : I \rightarrow \mathbb{R}$ be continuous on I, where I is a closed bounded interval. Then prove that for every $\varepsilon > 0$, there exists a step function $s_{\varepsilon} : I \rightarrow \mathbb{R}$ such that $|f(x) - s_{\varepsilon}(x)| < \varepsilon$ for all $x \in I$.

Answer **any three** from the following. Weight **3 each**.

21. State and prove the nested interval property.

22. If S is a subset of $\mathbb{R}$ that contain at least two points and has the property that $[x, y] \subseteq S$ whenever $x, y \in S$ and $x < y$, then show that S is an interval.

23. State and prove the Cauchy convergence criterion.

24. State and prove the Maximum-Minimum theorem.

25. If $I \subseteq \mathbb{R}$ is an interval and $f : I \rightarrow \mathbb{R}$ is monotone on I, then prove that the set of points $D \subseteq I$ at which f is discontinuous is a countable set.
