M 4442

Reg. No. : ....

Name : ....

V Semester B.A./B.Sc./B.Com./B.B.A./B.B.A. T.T.M./B.B.M./B.C.A./B.S.W./ B.A. Afsal-Ul-Ulama Degree (CCSS – Reg./Supple./Improve.) Examination, November 2013
Core Course in Mathematics
5B08MAT : GRAPH THEORY

Time : 3 Hours Max. Weightage : 30

***Instruction** : Answer to **all** questions.*

Fill in the blanks.

1. a) The complete bipartite graph $K_{4,\,6}$ has __________ number of edges.

   b) The complete graph K_4 has __________ number of spanning trees.

   c) The complete graph K_4 has __________ different Hamiltonian cycles.

   d) The order of the incidence matrix M(G) of a graph G with n vertices and t edges is **(4×¼=1 Wt.)**

Answer **any six** from the following. **(Wt. 1 each)**

2. When a simple graph G is S and to be self-complementary ? Give an example for a self complementary graph.

3. Draw the join of the graphs K_2 and K_3.

4. Where an edge e of a graph G is said to be contracted ? Illustrate it with an example.

5. Define a maximal spanning tree.

6. Define Euler and Hamiltonian graphs.

P.T.O.

7. Give an example of a matching in G which is maximum but not perfect.

8. When a digraph D is said to be K-regular ? Give an example for a 2-regular digraph.

9. Draw a (3, 2) de Bruiju diagram and use it to construct a (3, 2) de Bruiju sequence.

10. Prove that if a tournament T is Hamiltonian then it is strongly connected.

Answer **any seven** of the following. **(Wt. 2 each)**

11. Define the complement G of a simple graph G. Give an example for a self-complementary graph.

12. Find the radius and diameter of the Peterson graph.

13. Draw the graph having the following matrix $\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 2 \\ 2 & 3 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix}$ as its adjacency matrix.

14. Let G be a connected graph. Then prove that G is a tree if and only if every edge of G is bridge.

15. Let G be a connected graph.

    a) If G has 17 edges what is the maximum possible number of vertices in G ?

    b) If G has 21 vertices what is the minimum possible number of edges in G ?

    Justify your answer.

16. Prove that a simple graph G is Hamiltonian iff its closure is Hamiltonian.

17. Prove that a connected graph G has an euler trail if and only if it has at most two odd vertices.

18. Prove that a matching M in a graph G is a maximum matching if and only if G contains no M-augmenting path.

19. Prove that every tournament T has a directed Hamiltonian path.

20. Prove that for each pair of positive integer n and k, both greater than one, the de Bruiju diagram Dn, k has a directed Euler tour.

Answer **any three** of the following **(Wt. 3 each)**

21. Let G be a non empty graph with at least two vertices. Then prove that G is bipartite if and only if it has no odd cycles.

22. Let e be an edge of the graph G and as usual, let G-e be the subgraph obtained by deleting e. Then prove that $W(G) \leq W(G - e) \leq W(G) + 1$.

23. Let G be a simple graph with at least three vertices. Then prove that G is 2-connected if and only if for each pair of distinct vertices u and v of G these are internally two disjoint u-v paths in G.

24. If G is a simple graph with n vertices, where $n \geq 3$ and the degree $d(v) \geq n/2$ for every vertex V of G, then prove that G is Hamiltonian.

25. Prove that A graph G is orientable if and only if it is connected and has no bridges.