M 4443
Reg. No. : $\qquad$
Name: $\qquad$

# V Semester B.A./B.Sc./B.Com./B.B.A./B.B.A. T.T.M./B.B.M./B.C.A./B.S.W./ B.A. Afsal-Ul-ulama Degree (CCSS - Reg./Supple./Imp.) <br> Examination, November 2013 <br> Core Course in Mathematics <br> 5B09 MAT - Differential Equations and Numerical Analysis 

Time: 3 Hours
Max. Weightage : 30

1. Fill in the blanks :
(Weightage 1)
a) The general solution of $y^{\prime \prime}+y=0$ is $\qquad$
b) If $y_{1}(x)$ and $y_{2}(x)$ are two solutions of the equation $y^{\prime \prime}+P(x) y^{\prime}+Q(x) y=0$ on $[a, b]$ then they are linearly dependent on this interval if Wronskian is
c) The characteristic equation of $y^{\prime \prime}-y^{\prime}-6 y=0$ is $\qquad$
d) Consider the non homogeneous equation $y^{\prime \prime}+p y^{\prime}+q y=R(x)$, where $R(x)=e^{a x}$. Then if $a$ is not a root of the auxiliary equation, the particular solution is $\qquad$
Answer any six from the following (Weightage 1 each) :
2. Check whether the equation $(\sin x \tan y+1) d x+\operatorname{cosec}^{2} y d y=0$ is exact and if so solve it.
3. Solve $(x-4) y^{4} d x-x^{3}\left(y^{2}-3\right) d y=0$.
4. Solve $x^{2} y^{\prime \prime}+2 x y^{\prime}-12 y=0$.
5. Solve $y^{\prime \prime}-2 y^{\prime}+12 x-10$.
6. What are assumptions in the derivation of one dimensional wave equation?
7. How many initial and boundary conditions are required to solve one - dimensional heat flow equation? If time derivative is zero, what will be its solution?
8. Explain the Picard's method of successive approximation.
9. Using Newton Raphson method, find the root of the equation $x+\log _{10} x=3.375$ correct to four significant figures.
10. Find a solution of $x^{3}+x-1=0$ by iteration.

Answer any seven from the following (Weightage 2 each)
11. Solve $\left(2 x^{2} y^{2}+y\right) d x+\left(3 x-x^{3} y\right) d y=0$.
12. Solve $\cos (x+y) d y=d x$.
13. Solve $(1+x)^{2} \frac{d^{2} y}{d x^{2}}+(1+x) \frac{d y}{d x}+y=\sin 2[\log (1+x)]$.
14. Show that $y=C_{1} x+C_{2} x^{2}$ is the general solution of $x^{2} y^{\prime \prime}-2 x y^{\prime}+2 y=0$ on any interval not containing zero and find the solution for which $y(1)=3$ and $y^{\prime}(1)=5$.
15. Find the general solution of $y^{\prime \prime}-2 y^{\prime}+5 y=25 x^{2}+12$.
16. Using method of separation of variables, solve the equation $4 \frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3 u$ subject to $u=3 e^{-y}-e^{-5 y}$ when $x=0$.
17. If a string of length $4 l$ is initially at rest in its equilibrium position and each of its points is given initial velocity V where $\mathrm{V}=\frac{\mathrm{cx}}{l}$ in $0<\mathrm{x}<2 l$

$$
=\frac{\mathrm{c}}{l}(4 l-\mathrm{x}) \text { in } 2 l<\mathrm{x}<4 l .
$$

Find the displacement of the string at any time.
18. Solve the system of equations $3 x+y-z=3 ; 2 x-8 y+z=-5 ; x-2 y+9 z=8$ using Gauss elimination method.
19. The following table gives corresponding values of $x$ and $y$. From the difference table express $y$ as function of $x$ :

| $x$ | $:$ | 0 | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :---: | :---: | :---: |
| $y$ | $:$ | 3 | 6 | 11 | 18 | 27 |

20. Using Picard's method, find a solution of $\frac{d y}{d x}=2 x(1+y)$ upto fourth approximation, when $\mathrm{y}(0)=0$.

Answer any three from the following (Weightage 3 each) :
21. Solve $3 e^{x \tan y d x}+\left(1+e^{x}\right) \sec ^{2} y d y=0$, given $y=\frac{\pi}{4}$ when $x=0$.
22. Solve $\cos (x+y) d y=d x$.
23. Find a particular solution of $y^{\prime \prime}-y^{\prime}-6 y=e^{-x}$, first by undetermined coefficients and then by variation of parameters.
24. Given $y^{\prime}=x^{2}-y, y(0)=1$, find $y(0.1), y(0.2)$ using Runge-Kutta method of fourth order.
25. Using Taylor series method solve $\frac{d y}{d x}=x^{2}-y, y(0)=1$ at $x=0.1,0.2,0.3$ and 0.4 .

