M 4441

Reg. No. : ....

Name : ....

V Semester B.A./B.Sc./B.Com./B.B.A./B.B.A. T.T.M./B.B.M./B.C.A./B.S.W./ B.A. Afsal ul Ulama Degree (CCSS – Reg./Supple./Improv.) Examination, November 2013
Core Course in Mathematics
5B07 MAT : ABSTRACT ALGEBRA

Time : 3 Hours　　　　　　　　　　　　　　　　　　　　Max. Weightage : 30

1. Mark **each** of the following are **true** or **false** : **(Weightage 1)**
   a) The set Z^+ under addition is a group.
   b) Every group is a subgroup of itself.
   c) A subgroup of a cyclic group is cyclic.
   d) Q under addition is a cyclic group.

Answer **any 6** from the following **9** questions : **(Weightage 1 each)**

2. Define a group. Let $*$ be defined on R^* of nonzero real numbers by letting $a * b = a / b$. Is R^* a group under the operation $*$. Justify your answer.
3. Define a cyclic group. Find the number of generators of a cyclic group having the order 8.
4. Express the following permutation of {1, 2, 3, 4, 5, 6 7, 8} as a product of disjoints cycles and then as a product of transpositions.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 2 & 6 & 3 & 7 & 4 & 5 & 1 \end{pmatrix}$$

5. Let H be a subgroup of a group G and let $a \in G$. Define the left and right cosets of H containing a . Exhibit all left and right cosets of the subgroup 4Z of Z.
6. Define a group homomorphism. Compute Ker(ϕ) and $\phi(18)$ for $\phi: Z \to Z_{10}$ such that $\phi(1) = 6$.
7. Determine the number of group homomorphisms from Z into Z_2.

P.T.O.

8. Define the characteristic of a ring R. Find the characteristic of the ring $Z_6 \times Z_{15}$.

9. Define subring of a ring. Show that $2Z \cup 3Z$ is not a subring of Z.

10. Show that 1 and p -1 are the only elements of the field Z_p that are their own multiplicative inverse.

Answer **any 7** from the following **10** questions **(Weightage 2 each)**

11. Let G be an abelian group and let $c^n = c * c * \dots * c$ for n factors, where $c \in G$ and $n \in Z^+$. Then prove that $(a * b)^n = a^n * b^n$ for all integers n.

12. Show that if $a \in G$, where G is a finite group with identity e, then there exists $n \in Z^+$ such that $a^n = e$.

13. Define a subgroup of a group G. Find all subgroups of Z_{12}.

14. Show that if H is a subgroup of index 2 in a finite group G, then every left coset of H is also a right coset of H.

15. Let G be group of order pq, where p and q are prime numbers. Show that every proper subgroup of G is cyclic.

16. Describe the center of every simple.

    a) abelian group.

    b) nonabelian group.

17. Define normal subgroup of a group G. Show that the center Z(G) of a group G is a normal subgroup of G.

18. Let $\phi : G \to G'$ be a group homomorphism and let N be a normal subgroup of G. Then show that $\phi[N]$ is normal subgroup of $\phi[G]$.

19. If p is a prime, then show that Z_p is a field.

20. State and prove the Little Theorem of Fermat.

Answer **any 3** from the following **5** questions. **(Weightage 3 each)**

21. Show that Z_p has no proper nontrivial subgroups if p is a prime number.

22. Show that the order of an element of a finite group divides the order of the group.

23. Prove that every group is isomorphic to a group of permutations.

24. Show that M is a maximal normal subgroup of G if and only if G/M is simple.

25. Show that the set G_n of nonzero elements of Z_n that are not 0 divisors forms a group under multiplication modulo n.
