Reg. No. : ....

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V Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W./ B.A. Afsal Ul Ulama Degree (CCSS-Reg./Supple./Improv.) Examination, November 2012

CORE COURSE IN MATHEMATICS

5B05 MAT : Vector Analysis

Time: 3 Hours | Max. Weightage: 30

1. Fill in the blanks :

   a) Standard equation of a sphere of radius a and center (x_0, y_0, z_0) is ________

   b) If $f(x, y) = y \sin(xy)$, then $\frac{\partial f}{\partial y}$ = ________

   c) Cylindrical form of volume element dV is ________

   d) Formula for the flux of a three dimensional vector field $\vec{F}$ across an oriented surface S in the direction of $\hat{n}$ is ________ **(Weightage 1)**

Answer **any six** from the following. (Weightage **1 each**) :

2. Write the relation between rectangular and cylindrical co-ordinates.

3. Show that $\bar{u}(t) = \sin t\,\hat{i} + \cos t\,\hat{j} + \sqrt{3}\,\hat{k}$ is orthogonal to its derivative.

4. If $f(x, y) = xy + \frac{e^y}{y^2 + 1}$, find $\frac{\partial^2 f}{\partial x \partial y}$.

5. Find $\frac{dw}{dt}$ if $w = xy + z$, $x = \cos t$, $y = \sin t$ and $z = t$.

6. Find the gradient of $f(x, y) = y - x$ at the point (2, 1).

7. Find the area of the region R bounded by $y = x$ and $y = x^2$ in the first quadrant.

8. Find the average value of $f(x, y) = \sin(x + y)$ over the rectangle $0 \le x \le \pi, 0 \le y \le \pi$.

9. Find the circulation of the field $\vec{F} = (x - y)\hat{i} + x\hat{j}$ around the circle $\vec{r}(t) = \cos t\,\hat{i} + \sin t\,\hat{j}, 0 \le t \le 2\pi$.

10. State Green's theorem in plane. **(Weightage 6×1=6)**

Answer **any seven** from the following. (Weightage **2 each**) :

11. Find the vector projection of $\vec{b} = 6\hat{i} + 3\hat{j} + 2\hat{k}$ onto $\vec{a} = \hat{i} - 2\hat{j} - 2\hat{k}$ and scalar component of $\vec{b}$ in the direction of $\vec{a}$.

12. Find a vector perpendicular to the plane of P(1, –1, 0), Q(2, 1, –1) and R(–1, 1, 2).

13. Find the curvature of the helix $\vec{r}(t) = a\cos t\hat{i} + a\sin t\,\hat{j} + bt\,\hat{k}$, $a, b \ge 0, a^2 + b^2 \ne 0$.

14. Show that the function $f(x, y) = \dfrac{2x^2y}{x^4 + y^2}$ has no limit as (x, y) approaches (0, 0).

15. Find the linearization of $f(x, y) = (x + y + 2)^2$ at the point (1, 2).

16. Find the tangent plane and normal line of the surface $f(x, y, z) = x^2 + y^2 + z - 9 = 0$ at the point (1, 2, 4).

17. Evaluate $\iint\limits_R e^{x^2+y^2}\,dy\,dx$ where R is the semi-circular region bounded by the x-axis and the curve $y = \sqrt{1 - x^2}$.

18. Find the average value of xyz over the cube bounded by the coordinate planes and the planes x = 2, y = 2 and z = 2 in the first octant.

19. Show that $\vec{F} = \left(e^x \cos y + yz\right)\hat{i} + \left(xz - e^x \sin y\right)\hat{j} + (xy + z)\hat{k}$ is conservative and find a potential for it.

20. Find a parametrization of the sphere $x^2 + y^2 + z^2 = a^2$. **(Weightage 7×2=14)**

Answer **any three** from the following. (Weightage **3 each**) :

21. Find absolute maximum and minimum values of $f(x, y) = 2 + 2x + 2y - x^2 - y^2$ on the triangular plate in the first quadrant bounded by the lines $x = 0$, $y = 0$, $y = 9 - x$.

22. Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

23. Evaluate $\int_0^1 \int_0^{1-x} \sqrt{x+y}\,(y-2x)^2\,dy\,dx$.

24. Find the circulation of the field $\vec{F} = (x^2 - y)\hat{i} + 4z\,\hat{j} + x^2\,\hat{k}$ around the curve C in which the plane $z = 2$ meets the cone $z = \sqrt{x^2 + y^2}$, counterclockwise as viewed from above.

25. Verify divergence theorem for $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ over the rectangular parallelopiped, $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$. **(Weightage 3×3=9)**
