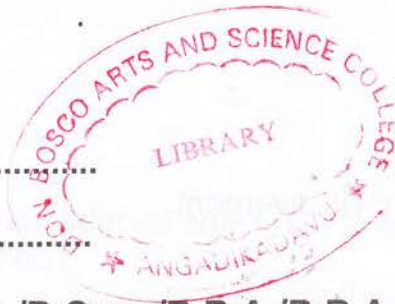




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Reg. No. : .....

Name : .....



V Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W./  
B.A. Afsal UI Ulama Degree (CCSS – Reg./Supple./Improv.)  
Examination, November 2012  
CORE COURSE IN MATHEMATICS  
5B06 MAT : Real Analysis

Time: 3 Hours

Max. Weightage : 30

1. Fill in the blanks :

a) If  $a$  is a real number and  $\varepsilon > 0$ , then the  $\varepsilon$ -neighbourhood of ' $a$ ' is \_\_\_\_\_

b)  $\sup \left\{ 1 - \frac{(-1)^n}{n} : n \in \mathbb{N} \right\} =$  \_\_\_\_\_

c) The set of all  $x \in \mathbb{R}$  that satisfy both the inequalities  $|2x - 3| < 5$  and  $|x + 1| > 2$  simultaneously is \_\_\_\_\_

d)  $\lim \left( \frac{\sqrt{n}}{n+1} \right) =$  \_\_\_\_\_

Answer any six from the following. Weight 1 each. (Wt. 1)

2. If  $a, b \in \mathbb{R}$ , prove that  $|a - b| \geq ||a| - |b||$ .

3. State and prove triangle inequality.

4. Show that the sequence  $\left( \frac{2n+1}{n} \right)$  converges to 2.

5. If  $X = (x_n)$  is a convergent sequence of real numbers and if  $x_n \geq 0$  for all  $n \in \mathbb{N}$ , then show that  $x = \lim(x_n) \geq 0$ .

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6. Show that the sequence  $((-1)^n)$  is divergent.
7. State the Raabe's test and show that  $\sum_{n=1}^{\infty} \left( \frac{n}{n^2 + 1} \right)$  is divergent.
8. State the Dirichlet test and Abel's test.
9. If  $f : A \rightarrow \mathbb{R}$  is uniformly continuous on a subset  $A$  of  $\mathbb{R}$  and if  $(x_n)$  is a Cauchy Sequence in  $A$ , then show that  $(f(x_n))$  is a Cauchy Sequence in  $\mathbb{R}$ .
10. If  $I$  is an interval and  $F : I \rightarrow \mathbb{R}$  is continuous on  $I$ , show that  $f(I)$  is an interval. **(6x1=6)**

Answer **any seven** from the following. Weight **2 each**.

11. State and prove the Archimedean property.
12. If  $x > -1$ , prove that  $(1 + x)^n \geq 1 + nx$  for all  $n \in \mathbb{N}$ .
13. Define the limit of a sequence in  $\mathbb{R}$ . Using the definition of limit show that 
$$\lim \left( \frac{3n+2}{n+1} \right) = 3.$$
14. Show that any convergent sequence of real numbers is a Cauchy Sequence.
15. If  $X = (x_n)$  and  $Y = (y_n)$  are sequence of real numbers that converge to  $x$  and  $y$  respectively. Show that  $X \cdot Y$  converges to  $x \cdot y$ .
16. Show that the series  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$  is convergent.
17. State the Dirichlet test for infinite series and using this prove that  $\sum_{n=1}^{\infty} \frac{1}{n} \cos nx$ , provided  $x \neq 2K\pi$ ,  $K \in \mathbb{N}$ , converges.
18. IF  $I$  is a closed bounded interval and  $f : I \rightarrow \mathbb{R}$  is continuous on  $I$ , then show that  $f(I) = \{f(x) : x \in I\}$  is a closed bounded interval.



- 19. If  $I$  is a closed bounded interval and  $f : I \rightarrow \mathbb{R}$  is continuous on  $I$ , show that  $f$  is uniformly continuous on  $I$ .
  - 20. Let  $f : I \rightarrow \mathbb{R}$  be continuous on  $I$ , where  $I = [a, b]$  is a closed bounded interval. If  $k \in \mathbb{R}$  satisfy  $\inf f(I) \leq k \leq \sup f(I)$ , then show that there exists some  $c \in I$  such that  $f(c) = k$ . (7x2=14)
- Answer **any three** from the following. Weight **3 each**.
- 21. Show that there exists a real number  $X$  such that  $x^2 = 2$ .
  - 22. State and prove the density theorem.
  - 23. If  $X = (x_n)$  is a sequence of real numbers, then prove that there is a subsequence of  $X$  that is monotone.
  - 24. Let  $I$  be a closed bounded interval and let  $f : I \rightarrow \mathbb{R}$  be continuous on  $I$ . Then prove that, for any  $\varepsilon > 0$ , there exists a continuous piecewise linear function  $g_\varepsilon : I \rightarrow \mathbb{R}$  such that  $|f(x) - g_\varepsilon(x)| < \varepsilon$  for all  $x \in I$ .
  - 25. If  $I = [a, b]$  is a closed bounded interval and if  $f : I \rightarrow \mathbb{R}$  is continuous on  $I$ , then prove that  $f$  has an absolute maximum and an absolute minimum on  $I$ . (3x3=9)