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Reg. No. : ....

Name : ....

V Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W./ B.A. Afsal Ul Ulama Degree (CCSS – Reg./Supple./Improv.) Examination, November 2012

CORE COURSE IN MATHEMATICS

5B06 MAT : Real Analysis

Time: 3 Hours Max. Weightage : 30

1. Fill in the blanks :

a) If a is a real number and $\varepsilon > 0$, then the ε-neighbourhood of 'a' is ___________

b) $\text{Sup}\left\{1-\frac{(-1)^n}{n} : n \in \mathbb{N}\right\} =$ ___________

c) The set of all $x \in \mathbb{R}$ that satisfy both the inequalities $|2x-3| < 5$ and $|x+1| > 2$ simultaneously is ___________

d) $\lim\left(\sqrt{n}/_{n+1}\right) =$ ___________

Answer **any six** from the following. Weight 1 **each**. **(Wt. 1)**

2. If a, $b \in \mathbb{R}$, prove that $|a-b| \geq \big||a|-|b|\big|$.

3. State and prove triangle inequality.

4. Show that the sequence $\left(\frac{2n+1}{n}\right)$ converges to 2.

5. If $X = (x_n)$ is a convergent sequence of real numbers and if $x_n \geq 0$ for all $n \in \mathbb{N}$, then show that $x = \lim(x_n) \geq 0$.

6. Show that the sequence $((-1)^n)$ is divergent.

7. State the Raabe's test and show that $\sum_{n=1}^{\infty}\left(\frac{n}{n^2+1}\right)$ is divergent.

8. State the Dirichlet test and Abel's test.

9. If $f : A \to \mathbb{R}$ is uniformly continuous on a subset A of $\mathbb{R}$ and if (x_n) is a Cauchy Sequence in A, then show that $(f(x_n))$ is a Cauchy Sequence in $\mathbb{R}$.

10. If I is an interval and $F : I \to \mathbb{R}$ is continuous on I, show that f(I) is an interval. **(6×1=6)**

Answer **any seven** from the following. Weight **2 each**.

11. State and prove the Archimedean property.

12. If $x > -1$, prove that $(1 + x)^n \geq 1 + nx$ for all $n \in \mathbb{N}$.

13. Define the limit of a sequence in $\mathbb{R}$. Using the definition of limit show that $\lim\left(\frac{3n+2}{n+1}\right) = 3$.

14. Show that any convergent sequence of real numbers is a Cauchy Sequence.

15. If $X = (x_n)$ and $Y = (y_n)$ are sequence of real numbers that converge to x and y respectively. Show that $X \cdot Y$ converges to $x \cdot y$.

16. Show that the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}$ is convergent.

17. State the Dirichlet test for infinite series and using this prove that $\sum_{n=1}^{\infty}\frac{1}{n}$ cosnx, provided $x \neq 2K\pi$, $K \in \mathbb{N}$, converges.

18. IF I is a closed bounded interval and $f : I \to \mathbb{R}$ is continuous on I, then show that $f(I) = \{f(x) : x \in I\}$ is a closed bounded interval.

19. If I is a closed bounded interval and $f : I \to \mathbb{R}$ is continuous on I, show that f is uniformly continuous on I.

20. Let $f : I \to \mathbb{R}$ be continuous on I, where $I = [a, b]$ is a closed bounded interval. If $k \in \mathbb{R}$ satisfy $\inf f(I) \leq k \leq \sup f(I)$, then show that there exists some $\mathbb{C} \in I$ such that $f(c) = K$.

(7×2=14)

Answer **any three** from the following. Weight **3 each.**

21. Show that there exists a real number X such that $x^2 = 2$.

22. State and prove the density theorem.

23. If $X = (x_n)$ is a sequence of real numbers, then prove that there is a subsequence of X that is monotone.

24. Let I be a closed bounded interval and let $f : I \to \mathbb{R}$ be continuous on I. Then prove that, for any $\varepsilon = 0$, there exists a continuous piecewise linear function $g_\varepsilon : I \to \mathbb{R}$ such that $|f(x) - g_\varepsilon(x)| < \varepsilon$ for all $x \in I$.

25. If $I = [a, b]$ is a closed bounded interval and if $f : I \to \mathbb{R}$ is continuous on I, then prove that f has an absolute maximum and an absolute minimum on I. **(3×3=9)**
