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Reg. No. : ....

Name : ....

V Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W./
B.A. Afsal Ul Ulama Degree (CCSS – Reg./Supple./Improv.)
Examination, November 2012
CORE COURSE IN MATHEMATICS
5 B08 MAT : Graph Theory

Time : 3 Hours Total Weightage : 30

***Instruction** : Answer to **all** questions.*

Fill in the blanks :

1. a) The number of vertices in the graph $K_{3,3}$ is __________

   b) The complete graph K_5 has __________ number of spanning trees.

   c) The number of edges in a 4-regular graph of 6 vertices is __________

   d) The order of the adjacency matrix of a graph G with n vertices is ______ **(Wt. 1)**

Answer **any six** from the following . (**Wt.1 each**) :

2. Define radius and diameter of a connected graph G.
3. Define the join of two graphs G_1 and G_2 with no vertex in common.
4. Draw a graph which is both Euler and Hamiltonian.
5. Give an example of a matching in G which is maximum but not perfect.
6. Give two examples for complete bipartite graphs which are also star graphs.
7. Define a digraph isomorphism.
8. Draw the $D_{2,3}$ Bruijn diagram and use it to construct a(2, 3) de Bruijn sequence.
9. Define a tournament and give an example.
10. When graph G is said to be randomly traceable for a vertex θ ? Give examples.

(6×1=6)

P.T.O.

Answer **any seven** of the following. Wt. **2 each** :

11. When a simple graph G is said to be self-complementary and give an example for a self-complementary graph.

12. Define the distance between two vertices u and v in a graph G. Prove that for any vertices u, v and w in G we have $d(u, w) \leq d(u, v) + d(v, w)$.

13. Prove that the graph G with adjacency matrix

$$A(G) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$ is connected.

14. Let G be an acyclic graph with n vertices and K connected components. Then prove that G has (n – k) edges.

15. Let G be a connected graph. Then prove that G is a tree if and only if every edge of G is a bridge.

16. State the travelling salesman problem.

17. Prove that a matching M in a graph G is a maximum matching if and only if G contains no M-augmenting paths.

18. Prove that a tournament T is Hamiltonian if and only if it is strongly connected.

19. Prove that every tournament T has a directed Hamiltonian path.

20. When a digraph D is s and to be unilaterally connected ? Give an example for a digraph which is unilaterally connected but not strongly connected. **(7×2=14)**

Answer **any three** of the following. Wt. **3 each** :

21. Let G be a graph with n vertices. Then prove that the following three statements are equivalent :

    i) G is a tree.

    ii) G is an acyclic graph with $(n - 1)$ edges.

    iii) G is a connected graph with $(n - 1)$ edges.

22. If G is a simple graph with n vertices where $n \geq 23$ and degree $d(v) \geq n/2$ for every vertex v of G, then prove that G is Hamiltonian.

23. Prove that a matching M in a graph G is a maximum matching if an only if G contains no M-augmenting path.

24. Let D be a weakly connected digraph with at least one arc. There prove that D is Euler if and only if $od(v) = id(v)$ for every vertex v of D.

25. Prove that a graph G is orientable if and only if it is connected and has no bridges.

(3×3=9)