M 1988

Reg. No. : ....

Name : ....

V Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W./ B.A. Afsal Ul Ulama Degree (CCSS – Reg./Supple./Improv.) Examination, November 2012

CORE COURSE IN MATHEMATICS

5B09 MAT : Differential Equations and Numerical Analysis

Time: 3 Hours   Max. Weightage : 30

1. Fill in the blanks :   **(Weightage : 1)**

   a) Wronskian of y_1 and y_2 $W[y_1, y_2]$ is ____________

   b) Let m_1 and m_2 be the roots of the characteristic equation of $y'' + py' + qy = 0$. If m_1 and m_2 are real and equal say m, then the general solution is ________

   c) The characteristic equation of $y'' - 2y' - 3y = 0$ is ____________

   d) Consider the non homogeneous equation $y'' + py' + qy = R(x)$, where $R(x) = e^{ax}$. Then if a is a double root of the auxillary equation, the particular solution is ____________

Answer **any six** from the following (Weightage **1 each)** :

2. Solve $e^y + (xe^y + 2y)\, dy = 0$.

3. Determine the order of the differential equation $\frac{d^2y}{dx^2} - \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = 0$.

4. Find the general solution of $y'' + 2y' + y = 0$.

5. Find the particular solution of $y'' - 4y = \tan x$ by the method of variation of parameters.

6. A rod of length l is heated so that the ends A and B are at zero temperature. If initially its temperature is given by $u = \frac{cx(l - x)}{l^2}$. What are the boundary conditions.

P.T.O.

7. What are the assumptions in the derivation of one dimensional wave equation ?

8. Explain the Taylor series method for solving the first order differential equation.

9. Find by Newton's method, the real root of the equation $xe^x - 2 = 0$ correct to two decimal places.

10. Find the cubic polynomial which takes the following values :

X	0	1	2	3
f(x)	1	2	1	10

(6×1=6)

Answer **any seven** from the following (Weightage **2 each**) :

11. Solve $\left(xy^2 - e^{x^{\frac{1}{3}}}\right)dx - x^2y\,dy = 0$.

12. Find the solution of the differential equation $\frac{dy}{dx} - x\tan(y - x) = 1$.

13. Solve $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4$.

14. Show that $y = C_1e^x + C_2e^{2x}$ is the general solution of $y'' - 3y' + 2y = 0$ on any interval and find the particular solution for which $y(0) = -1$ and $y'(0) = 1$.

15. Find the general solution of $y'' + y = 2\cos x$.

16. Solve the following equation using the method of separation of variables.

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u,\ u(x, 0) = 6e^{-3x}.$$

17. A rod of length l has its ends A and B maintained at 0°C and 100°C respectively, until steady – state conditions prevail. If B is suddenly reduced to 0°C and kept so, while that of A is maintained, find the temperature function u(x, t).

18. Using Newton's forward interpolation formula, find y at x = 8 from the following table :

x :	0	5	10	15	20	25
y :	7	11	14	18	24	32

19. Apply Gauss elimination method of solve the equations $x + 4y - z = 5$, $x - y - 6z = -12$; $3x - y - z = 4$.

20. Using Picard's method, find a solution of $\frac{dy}{dx} = x + y$ upto fourth approximation, when $y(0) = 1$. **(7×2=14)**

Answer **any three** from the following (Weightage **3 each**) :

21. Solves $\cos(x + y)dy = dx$.

22. Solve $(x^2 - y^2)\, dx = 2xydy$.

23. Solve by the method of variation of parameters

$$(x^2 + x)\, y'' + (2 - x^2)y' - (2 + x)y = x(x + 1)^2.$$

24. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using

    i) Simpson's one third rule taking $h = \frac{1}{4}$,

    ii) Simpson's three eighth rule taking $h = \frac{1}{6}$.

25. Using Runge-Kutta method of fourth order solve for y(0.1), y(0.2) and y(0.3) given that $y' = xy + y^2, y(0) = 1$. **(3×3=9)**

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