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Reg. No. : ....

Name : ....

V Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W./ B.A. Afsal Ul Ulama Degree (CCSS – Reg./Supple./Improv.) Examination, November 2012
CORE COURSE IN MATHEMATICS
5B07 MAT : Abstract Algebra

Time: 3 Hours Max. Weightage : 30

1. Mark **each** of the following are **true** or **false** (Weightage **1**) :
   a) If $*$ is any binary operation on any set S, then $a*a = a$ for all $a \in G$.
   b) Any finite group of atmost three elements is abelian.
   c) Every cyclic group is abelian.
   d) Every group of order ≤ 4 is cyclic.

Answer **any 6** from the following **9** questions (Weightage **1 each**) :

2. Define a cyclic group. Find the number of generators of a cyclic group having the order 8.
3. Define a cyclic group. Find the number of generators of a cyclic group having the order 12.
4. Define orbit of a permutation. Find the orbits of the permutation.
$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 6 & 7 & 4 & 1 & 5 & 2 \end{pmatrix} \text{ in } S_8.$$
5. Let H be a subgroup of a group G and let $a \in G$. Define the left and right cosets of H containing a. Exhibit all left and right cosets of the subgroup 4Z of 2Z.
6. Define a group homomorphism. Determine whether the map $\phi : R^* \to R^*$ under multiplication given by $\phi(x) = |x|$ is an homomorphism.
7. Determine the number of group homomorphisms from Z into Z.
8. Define a ring. Give an example of a ring having two elements a and b such that ab = 0 but neither a nor b is zero.
9. Define the characteristic of a ring R. Find the characteristic of the ring $Z_3 \times Z_4$.
10. Define an idempotent element of a ring. Show that a division ring contains exactly two idempotent elements.

(6×1=6)

P.T.O.

Answer **any 7** from the following **10** questions (Weightage **2 each**) :

11. Prove that if G is an abelian group, written multiplicatively, with identity element e, then all elements x of G satisfying the equation $x^2 = e$ form a subgroup H of G.
12. Define a subgroup of a group G. Find all subgroups of Z_{18}.
13. Show that if H and K are subgroups of an abelian group G, then $\{hk \setminus h \in H \text{ and } k \in K\}$ is a subgroup of G.
14. Obtain the group of symmetries of an equilateral triangle with vertices 1, 2 and 3. Show that it is nonabelian.
15. Prove that a group G of prime order is cyclic.
16. Define normal subgroup of a group G. If $\phi : G \to G'$ is a group homomorphism, then show that Ker (ϕ) is a normal subgroup of G.
17. Show that A_n is a normal subgroup of S_n and find a known group to which S_n / A_n is isomorphic.
18. Show that if H and K are normal subgroups of a group G such that $H \cap K = \{e\}$, then $hk = kh$ for all $h \in H$ and $k \in K$.
19. Show that the characteristic of an integral domain D must be either 0 or a prime p.
20. Find all solutions of the congruence $15x \equiv 27 \pmod{18}$. **(7×2=14)**

Answer **any 3** from the following **5** questions (Weightage **3 each**) :

21. Show that a subgroup of a cyclic group is cyclic.
22. Prove that any permutation of a finite set of at least two elements is a product of transpositions.
23. Let H be a subgroup of G such that $g^{-1}hg \in H$ for all $g \in G$ and all $h \in H$. show that every left coset gH is the same as the right coset Hg.
24. State and prove the fundamental homomorphism theorem.
25. If $a \in Z$, the show that $a^p \equiv a \pmod{p}$ for any prime p. **(3×3=9)**
