M 11408

Reg. No. : ....

Name : ....

V Semester B.A./B.Sc./B.Com./B.B.A./B.B.A. T.T.M./B.B.M./B.C.A./B.S.W. Degree (CCSS – Regular) Examination, November 2011
CORE COURSE IN MATHEMATICS
5B06 MAT : Real Analysis

Time: 3 Hours　　　　Max. Weightage : 30

1. Fill in the blanks :

   a) If $a \in \mathbb{R}$ is such that $0 \le a < \varepsilon$ for every $\varepsilon > 0$, then a = ____________

   b) If x > –1, then $(1 + x)^n \ge$ ____________

   c) $\text{Sup}\left\{\frac{1}{n} - \frac{1}{m} : n, m \in \mathbb{N}\right\} =$ ____________

   d) The set of all $x \in \mathbb{R}$ that satisfy $|x^2 - 1| \le 3$ is ________　**(Wt. 1×1=1)**

Answer **any six** from the following. Weight **1 each** :

2. If a and b are rationals and $b \ne 0$ then show that $a + b\sqrt{2}$ is irrational.
3. For all $x, y \in \mathbb{R}$, prove that $|xy| = |x||y|$
4. Prove that the sequence $\left(\frac{1}{n}\right)$ converges to zero.
5. Show that a convergent sequence of real numbers is bounded.
6. Show that every convergent sequence $X = (x_n)$ of real numbers is a Cauchy sequence.
7. Prove that an absolutely convergent series in $\mathbb{R}$ is convergent.
8. State the integral test.
9. Define continuity and uniform continuity. Give an example of a function which is continuous but not uniformly continuous.
10. If $f : A \to \mathbb{R}$, $A \subseteq \mathbb{R}$, is a lipschitz function then show that f is uniformly continuous on A.

Answer **any seven** from the following. Weight **2 each** :

11. If $a, b \in \mathbb{R}$, prove that $||a|-|b|| \leq |a-b|$
12. Prove that the set of real numbers is not countable.
13. Show that a sequence in $\mathbb{R}$ can have at most one limit.
14. If $0 < b < 1$, show that $\lim (b^n) = 0$.
15. Define a contractive sequence and prove that every contractive sequence is a Cauchy sequence.
16. Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent.
17. State and prove that Abel's test for infinite series.
18. Let I be an interval and $f : I \to \mathbb{R}$ be continuous on I. Let $a, b \in I$ and $k \in \mathbb{R}$ satisfies $f(a) < k < f(b)$. Then prove that there exists a point $C \in I$ between a and b such that $f(c) = k$.
19. If I is an interval and $f : I \to \mathbb{R}$ is continuous then prove that f(I) is an interval.
20. Let $f : I \to \mathbb{R}$ be an increasing function on I, where $I \subseteq \mathbb{R}$ is an interval. If $C \in I$ and C is not an end point of I, then prove that $\lim_{x \to c} - f = \text{Sup} \{f(x) : x \in I, x < c\}$.

Answer **any three** from the following. Weight **3 each** :

21. If S is a non-empty subset of $\mathbb{R}$, prove that Sup $(a + S) = a +$ Sup S, where $a \in \mathbb{R}$.
22. State and prove the density theorem.
23. State and prove the monotone convergence theorem.
24. State and prove the maximum-minimum theorem.
25. Let $I \subseteq \mathbb{R}$ be an interval and let $f : I \to \mathbb{R}$ be strictly monotone and continuous on I. Then prove that the function g inverse to f is strictly monotone and continuous on $J = f(I)$.
